thm_2Ereal__topology_2ECLOSURE__HALFSPACE__LT (TMQQkupsT-tWNT7jrak8gEMoqCzLxPRN4xnR)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (2)$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (3)$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (4)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (5)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (6)$$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 12 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 13 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 2$)\ (\lambda V2t \in$

Definition 14 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \quad (7)$$

Definition 15 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 16 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 17 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ ($c_2Ebool_2E

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (8)$$

Definition 18 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (10)$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (11)$$

Definition 19 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $ty\_2Epair\_2Eprod : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}\forall A1.nonempty\ A1{\Rightarrow}nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{12}$$

Definition 20 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 21 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Definition 22 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 23 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{13}$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{14}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{15}$$

Definition 24 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 (t

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehr} \tag{16}$$

Definition 25 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{17}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{18}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr} \tag{19}$$

Definition 26 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ t$

Definition 27 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \quad (20)$$

Definition 28 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Definition 29 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 30 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 31 *We define* c_2Epred__set_2EUNIV *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (21)$$

Definition 32 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \quad (22)$$

Definition 33 *We define* c_2Epred__set_2EDIFF *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2$

Let $c\_2Ereal\_\_topology\_2EDist : \iota$ be given. Assume the following.

$$c\_2Ereal\_\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \quad (23)$$

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (24)$$

Definition 34 *We define* c_2Ereal__topology_2EOpen *to be* $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap$ (c_2Ebool_2E_2

Definition 35 *We define* c_2Ereal__topology_2Elimit__point__of *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1s \in ($

Definition 36 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c.$

Definition 37 *We define* c_2Ereal__topology_2Eclosure *to be* $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap\ (ap$ (c_2Epred

Definition 38 *We define* c_2Ereal_2Ereal__gt *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ere$

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \tag{25}$$

Definition 39 *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

Definition 40 *We define* c_2Ereal_2Ereal__ge *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 41 *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ ($

Definition 42 *We define* c_2Ereal__topology_2Einterior *to be* $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap\ (c\_2Epred\_\_s$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Enum\_2E0})\ V0n))) \tag{26}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((\neg(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1n)\ V0m))))) \tag{27}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = \mathsf{c\_2Enum\_2E0})\Leftrightarrow((V0m = \mathsf{c\_2Enum\_2E0})\wedge(V1n = \mathsf{c\_2Enum\_2E0}))))) \tag{28}$$

Assume the following.

$$((\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow(V0n = \mathsf{c\_2Enum\_2E0})))\wedge(\forall V1m \in ty\_2Enum\_2Enum.(\forall V2n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V2n)))\Leftrightarrow((V1m = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V2n))\vee(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1m)\ V2n))))))) \tag{29}$$

Assume the following.

$$True \tag{30}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{31}$$

Assume the following.

$$(\forall V0t \in 2.(False\Rightarrow(p\ V0t))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)){\Leftrightarrow}(p\ V0t))) \tag{33}$$

Assume the following.

$$(\forall V0t \in 2.(((True{\wedge}(p\ V0t)){\Leftrightarrow}(p\ V0t)){\wedge}((((p\ V0t){\wedge}True){\Leftrightarrow}(p\ V0t)){\wedge}(((False{\wedge}(p\ V0t)){\Leftrightarrow}False){\wedge}((((p\ V0t){\wedge}False){\Leftrightarrow}False){\wedge}(((p\ V0t){\wedge}(p\ V0t)){\Leftrightarrow}(p\ V0t))))))) \tag{34}$$

Assume the following.

$$(\forall V0t \in 2.(((True{\Rightarrow}(p\ V0t)){\Leftrightarrow}(p\ V0t)){\wedge}((((p\ V0t){\Rightarrow}True){\Leftrightarrow}True){\wedge}(((False{\Rightarrow}(p\ V0t)){\Leftrightarrow}True){\wedge}((((p\ V0t){\Rightarrow}(p\ V0t)){\Leftrightarrow}True){\wedge}(((p\ V0t){\Rightarrow}False){\Leftrightarrow}(\neg(p\ V0t)))))))) \tag{35}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))){\Leftrightarrow}(p\ V0t))){\wedge}(((\neg True){\Leftrightarrow}False){\wedge}((\neg False){\Leftrightarrow}True))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.((V0x = V0x){\Leftrightarrow}True)) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y){\Leftrightarrow}(V1y = V0x)))) \tag{38}$$

Assume the following.

$$(\forall V0t \in 2.(((True{\Leftrightarrow}(p\ V0t)){\Leftrightarrow}(p\ V0t)){\wedge}((((p\ V0t){\Leftrightarrow}True){\Leftrightarrow}(p\ V0t)){\wedge}(((False{\Leftrightarrow}(p\ V0t)){\Leftrightarrow}(\neg(p\ V0t))){\wedge}(((p\ V0t){\Leftrightarrow}False){\Leftrightarrow}(\neg(p\ V0t))))))) \tag{39}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A){\wedge}(p\ V1B))){\Leftrightarrow}((\neg(p\ V0A)){\vee}(\neg(p\ V1B)))){\wedge}((\neg((p\ V0A){\vee}(p\ V1B))){\Leftrightarrow}((\neg(p\ V0A)){\wedge}(\neg(p\ V1B))))))) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a){\wedge}(p\ (ap\ V0P\ V2x)))){\Leftrightarrow}(p\ (ap\ V0P\ V1a))))) \tag{41}$$

Assume the following.

$$
\begin{gathered}
((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
\mathsf{c\_2Enum\_2E0})\ V0n) = V0n)) \land ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
(ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ \mathsf{c\_2Enum\_2E0}) = V1n)) \land ((\forall V2n \in \\
ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V3m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap \\
(ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ V3m)))))) \land ((\forall V4n \in ty\_2Enum\_2Enum. \\
((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \mathsf{c\_2Enum\_2E0})\ V4n) = \mathsf{c\_2Enum\_2E0})) \land \\
((\forall V5n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A \\
V5n)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0})) \land ((\forall V6n \in ty\_2Enum\_2Enum. \\
(\forall V7m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (\\
ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V6n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V7m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A \\
V6n)\ V7m))))) \land ((\forall V8n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D \\
\mathsf{c\_2Enum\_2E0})\ V8n) = \mathsf{c\_2Enum\_2E0})) \land ((\forall V9n \in ty\_2Enum\_2Enum. \\
((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V9n)\ \mathsf{c\_2Enum\_2E0}) = V9n)) \land ((\forall V10n \in \\
ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D \\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V10n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V11m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D \\
V10n)\ V11m))))) \land ((\forall V12n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP \\
\mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
V12n))) = \mathsf{c\_2Enum\_2E0})) \land ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
(ap\ c\_2Earithmetic\_2EEXP\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V13n))) = \mathsf{c\_2Enum\_2E0})) \land ((\forall V14n \in \\
ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V14n)\ \mathsf{c\_2Enum\_2E0}) = \\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))) \land \\
((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V15n)) \\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V16m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(ap\ (ap\ c\_2Earithmetic\_2EEXP\ V15n)\ V16m))))) \land (((ap\ \mathsf{c\_2Enum\_2ESUC} \\
\mathsf{c\_2Enum\_2E0}) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
\mathsf{c\_2Earithmetic\_2EZERO}))) \land ((\forall V17n \in ty\_2Enum\_2Enum.(\\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V17n)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ V17n)))) \land (((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ \mathsf{c\_2Enum\_2E0}) = \\
\mathsf{c\_2Enum\_2E0}) \land ((\forall V18n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE} \\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V18n)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ V18n)))) \land ((\forall V19n \in ty\_2Enum\_2Enum. \\
(((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V19n) = \mathsf{c\_2Enum\_2E0}) \Leftrightarrow (V19n = \mathsf{c\_2Earithmetic\_2EZERO}))) \land \\
((\forall V20n \in ty\_2Enum\_2Enum.((\mathsf{c\_2Enum\_2E0} = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V20n)) \Leftrightarrow (V20n = \mathsf{c\_2Earithmetic\_2EZERO}))) \land ((\forall V21n \in ty\_2Enum\_2Enum. \\
(\forall V22m \in ty\_2Enum\_2Enum.(((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V21n) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V22m)) \Leftrightarrow (V21n = V22m)))) \land \\
((\forall V23n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \\
V23n)\ \mathsf{c\_2Enum\_2E0})) \Leftrightarrow False)) \land ((\forall V24n \in ty\_2Enum\_2Enum. \\
((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V24n))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO}) \\
V24n)))) \land ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V25n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V26m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \\
V25n)\ V26m))))) \land ((\forall V27n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E} \\
\mathsf{c\_2Enum\_2E0})\ V27n)) \Leftrightarrow False)) \land ((\forall V28n \in ty\_2Enum\_2Enum. \\
((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V28n))\ \mathsf{c\_2Enum\_2E0})) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO}) \\
V28n)))) \land ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V29n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V30m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \\
V30m)\ V29n))))) \land ((\forall V31n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \\
\mathsf{c\_2Enum\_2E0})\ V31n)) \Leftrightarrow True)) \land ((\forall V32n \in ty\_2Enum\_2Enum. \\
((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}
\end{gathered}
$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2a \in A\_27a.(\forall V3b \in A\_27b.(((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))) \tag{44}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}).(\forall V1v \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1v)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V1v)\ \mathsf{c\_2Ebool\_2ET}) = (ap\ V0f\ V2x)))))) \tag{45}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ A\_27a)))) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDIFF}\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \wedge (\neg (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t)))))))) \tag{47}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. ((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V1y)\ V0x)))) \tag{48}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. (\forall V2z \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V1y)\ V2z)) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y))\ V2z))))) \tag{49}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0x) = V0x)) \tag{50}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x))\ V0x) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))) \tag{51}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(\forall V2z \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V0x)\ V1y)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V1y)\ V2z))) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V0x)\ V2z)))))) \tag{52}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V0x) = V0x)) \tag{53}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) = V0x)) \tag{54}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x)) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))) \tag{55}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0x) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))) \tag{56}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(\forall V2z \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V2z))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V1y)\ V2z)))))) \tag{57}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(\forall V2z \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V0x)\ V1y))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1y)\ V2z)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V0x)\ V2z)))))) \tag{58}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(\forall V2z \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ V1y))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V1y)\ V2z)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V0x)\ V2z)))))) \tag{59}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(\forall V2z \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ V1y))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1y)\ V2z)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ V2z)))))) \tag{60}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ V1y))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1y)\ V0x)))\Leftrightarrow(V0x = V1y)))) \tag{61}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ V0m))\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ V1n)) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))))) \tag{62}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x))\ V1y) = (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ V1y))))) \tag{63}$$

Assume the following.

$$(\forall V0y \in ty\_2Erealax\_2Ereal.(\forall V1x \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V1x)\ V0y))\Leftrightarrow(\neg(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0y)\ V1x)))))) \tag{64}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(\forall V2z \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1y)\ V2z)) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V2z))))))) \tag{65}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x))\ V1y)) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y)))))) \tag{66}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x))\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V1y))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1y)\ V0x))))) \tag{67}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x)) = V0x)) \tag{68}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V1y))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y))\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))))) \tag{69}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(\forall V2z \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y))\ V2z) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ V2z))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V1y)\ V2z)))))) \tag{70}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ V0m))\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n))))) \tag{71}$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).((ap\ \mathsf{c\_2Ereal\_\_topology\_2Eclosure}\ V0s) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDIFF}\ ty\_2Erealax\_2Ereal)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ ty\_2Erealax\_2Ereal))\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Einterior}\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDIFF}\ ty\_2Erealax\_2Ereal)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ ty\_2Erealax\_2Ereal))\ V0s))))) \tag{72}$$

Assume the following.

$$(\forall V0a \in ty\_2Erealax\_2Ereal.(\forall V1b \in ty\_2Erealax\_2Ereal.((\neg(V0a = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\Rightarrow((ap\ \mathsf{c\_2Ereal\_\_topology\_2Einterior}\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ (\lambda V2x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ 2)\ V2x)\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_ge}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0a)\ V2x))\ V1b))))) = (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ (\lambda V3x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ 2)\ V3x)\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_gt}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0a)\ V3x))\ V1b)))))))) \tag{73}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{74}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{75}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow(((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{76}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{77}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{78}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee((\neg(p\ V1q))\vee(\neg(p\ V0p))))))))))) \tag{79}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee(\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{80}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{81}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{82}$$

Theorem 1

$$(\forall V0a \in ty\_2Erealax\_2Ereal.(\forall V1b \in ty\_2Erealax\_2Ereal.((\neg(V0a = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\Rightarrow((ap\ \mathsf{c\_2Ereal\_\_topology\_2Eclosure}\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ (\lambda V2x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ 2)\ V2x)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0a)\ V2x))\ V1b))))) = (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ (\lambda V3x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ 2)\ V3x)\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0a)\ V3x))\ V1b))))))))$$