

thm_2Ereal__topology_2ECLOSURE__IMAGE__BOUNDED (TMMQR2HL2dNhc15u3ei3ATeh1vwt8bjte1g)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \tag{1}$$

Definition 3 We define $c_2Epred_set_2EUNIV$ to be $\lambda A.27a : \iota.(\lambda V0x \in A.27a.c_2Ebool_2ET)$.

Definition 4 We define c_2Ebool_2EIN to be $\lambda A.27a : \iota.(\lambda V0x \in A.27a.(\lambda V1f \in (2^{A-27a}).(ap\ V1f\ V0x)))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 6 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21\ 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2EF))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Epair_2EABS_prod\ A.27a\ A.27b \in ((ty_2Epair_2Eprod\ A.27a\ A.27b)^{(2^{A-27b})^{A-27a}}) \tag{3}$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epred_set_2EGSPEC A.27a A.27b \in ((2^{A.27a})^{(ty_2Epair_2Eprod A.27a 2)^{A.27b}})$$
(4)

Definition 11 We define $c_2Epred_set_2EDIFF$ to be $\lambda A.27a : \iota.\lambda V0s \in (2^{A.27a}).\lambda V1t \in (2^{A.27a}).(ap (c_2Ereal_topology_2EDist : \iota)$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)})$$
(5)

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty ty_2Ehreal_2Ehreal$$
(6)

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal})$$
(7)

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$.

Definition 13 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (t$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)})$$
(8)

Definition 14 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega$$
(9)

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum$$
(10)

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega})$$
(11)

Definition 15 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum})$$
(12)

Definition 16 We define c_Ebool_E3F to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_Emin_E40$

Definition 17 We define $c_Ereal_topology_EOpen$ to be $\lambda V0s \in (2^{ty_Erealax_Ereal}). (ap (c_Ebool_E2$

Definition 18 We define $c_Ereal_topology_EClosed$ to be $\lambda V0s \in (2^{ty_Erealax_Ereal}). (ap c_Ereal_topo$

Let $c_Erealax_Etrealm_neg : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_neg \in ((ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal) (ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)) \quad (13)$$

Let $c_Erealax_Etrealm_eq : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_eq \in ((2^{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)}) (ty_Epair_Eprod ty_Ehreal_Ehreal)) \quad (14)$$

Let $c_Erealax_Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_Ereal_ABS_CLASS \in (ty_Erealax_Ereal)^{(2^{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)})} \quad (15)$$

Definition 19 We define $c_Erealax_Ereal_ABS$ to be $\lambda V0r \in (ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)$

Definition 20 We define $c_Erealax_Ereal_neg$ to be $\lambda V0T1 \in ty_Erealax_Ereal. (ap c_Erealax_Ereal.$

Definition 21 We define $c_Ereal_Ereal_lte$ to be $\lambda V0x \in ty_Erealax_Ereal. \lambda V1y \in ty_Erealax_Ereal.$

Definition 22 We define c_Ebool_ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 23 We define c_Ereal_Eabs to be $\lambda V0x \in ty_Erealax_Ereal. (ap (ap (ap (c_Ebool_ECOND$

Definition 24 We define $c_Ereal_topology_Ebounded_def$ to be $\lambda V0s \in (2^{ty_Erealax_Ereal}). (ap (c_Ebo$

Let $c_Eenum_EREP_num : \iota$ be given. Assume the following.

$$c_Eenum_EREP_num \in (\omega^{ty_Eenum_Eenum}) \quad (16)$$

Let $c_Eenum_ESUC_REP : \iota$ be given. Assume the following.

$$c_Eenum_ESUC_REP \in (\omega^{\omega}) \quad (17)$$

Definition 25 We define c_Eenum_ESUC to be $\lambda V0m \in ty_Eenum_Eenum. (ap c_Eenum_EABS_num$

Definition 26 We define $c_Eprim_rec_E3C$ to be $\lambda V0m \in ty_Eenum_Eenum. \lambda V1n \in ty_Eenum_Eenum.$

Definition 27 We define $c_Earithmetic_E3E$ to be $\lambda V0m \in ty_Eenum_Eenum. \lambda V1n \in ty_Eenum_Eenum.$

Definition 28 We define $c_Ebool_E5C_E2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_Ebool_E21 2) (\lambda V2t$

Definition 29 We define $c_Earithmetic_E3E_E3D$ to be $\lambda V0m \in ty_Eenum_Eenum. \lambda V1n \in ty_Eenum_Eenum.$

Assume the following.

$$(\forall V0t \in 2.(((p \ V0t) \Rightarrow False) \Rightarrow (\neg(p \ V0t)))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p \ V0t)) \Rightarrow ((p \ V0t) \Rightarrow False))) \quad (26)$$

Assume the following.

$$\begin{aligned} &(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\ &(p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\ &(((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} &(\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\ &(((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\ &(p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} &(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\ &True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\ &(p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} &((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ &((\neg False) \Leftrightarrow True))) \end{aligned} \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (31)$$

Assume the following.

$$\begin{aligned} &\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ &A_27a.(\forall V2z \in A_27a.(((V0x = V1y) \wedge (V1y = V2z)) \Rightarrow (V0x = V2z)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} &(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\ &(p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ &p \ V0t)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} &(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee (\\ &(p \ V1B) \vee (p \ V2C)) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C)))))) \end{aligned} \quad (34)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge ((\neg((\neg(p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B)))))))))) \quad (36)$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}).((p (ap c_2Ereal_topology_2EClosed V0s)) \Rightarrow ((ap c_2Ereal_topology_2Eclosure V0s) = V0s))) \quad (37)$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}).((p (ap c_2Ereal_topology_2Ecompact V0s)) \Rightarrow (p (ap c_2Ereal_topology_2EClosed V0s)))) \quad (38)$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}).((p (ap c_2Ereal_topology_2Ecompact (ap c_2Ereal_topology_2Eclosure V0s))) \Leftrightarrow (p (ap c_2Ereal_topology_2Ebounded_def V0s)))))) \quad (39)$$

Assume the following.

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in (2^{ty_2Erealax_2Ereal}).(((p (ap (ap c_2Ereal_topology_2Econtinuous_on V0f) V1s)) \wedge (p (ap c_2Ereal_topology_2Ecompact V1s))) \Rightarrow (p (ap c_2Ereal_topology_2Ecompact (ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0f) V1s))))))) \quad (40)$$

Assume the following.

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in (2^{ty_2Erealax_2Ereal}).((p (ap (ap c_2Ereal_topology_2Econtinuous_on V0f) (ap c_2Ereal_topology_2Eclosure V1s))) \Rightarrow ((ap c_2Ereal_topology_2Eclosure (ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0f) (ap c_2Ereal_topology_2Eclosure V1s))) = (ap c_2Ereal_topology_2Eclosure (ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0f) V1s))))))) \quad (41)$$

Assume the following.

$$(\forall V0t \in 2.(((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (44)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (45)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (50)$$

Theorem 1

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V1s \in (2^{ty_2Erealax_2Ereal}). (((p (ap (ap c_2Ereal_topology_2Econtinuous_on V0f) (ap c_2Ereal_topology_2Eclosure V1s))) \wedge (p (ap c_2Ereal_topology_2Ebounded_def V1s))) \Rightarrow ((ap c_2Ereal_topology_2Eclosure (ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0f) V1s)) = (ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0f) (ap c_2Ereal_topology_2Eclosure V1s))))))$$