

thm_2Ereal__topology_2ECLOSURE__SEQUENTIAL (TMd5CaYYvtiAFRkKYKCSvPmbuayEgTmyHmN)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ecombin_2EK$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.(\lambda V0x \in A.\lambda V1y \in A.\lambda V0x \in A.\lambda V1y \in A.V0x)$

Definition 3 We define $c_2Ecombin_2ES$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda A.\lambda c : \iota.(\lambda V0f \in ((A.\lambda c^{A-27b})^{A-27a}))$

Definition 4 We define $c_2Ecombin_2EI$ to be $\lambda A.\lambda a : \iota.(ap (ap (c_2Ecombin_2ES A.\lambda a (A.\lambda a^{A-27a})) A.\lambda a))$

Definition 5 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))))$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2)) (\lambda V2t \in 2.V2t)))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda A.\lambda c : \iota.(ap (ap (c_2Epair_2EABS_prod A.\lambda a A.\lambda b) ((ty_2Epair_2Eprod A.\lambda a A.\lambda b)^{(2^{A-27b})^{A-27a}})) A.\lambda c) \quad (2)$$

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda V0x \in A.\lambda V1y \in A.\lambda V2z \in A.(ap (c_2Epair_2EABS_prod A.\lambda a A.\lambda b) ((ty_2Epair_2Eprod A.\lambda a A.\lambda b)^{(2^{A-27b})^{A-27a}})) A.\lambda V2z)$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}})$$
(3)

Definition 10 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Definition 11 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (ap\ V2t\ t1))))$

Definition 12 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Epred_set_2EGSPEC\ A_27a\ s)\ V1t)$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal$$
(4)

Definition 13 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2. V0t))$.

Definition 14 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_21\ 2))$

Definition 15 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x. x \in A \wedge p\ (ap\ P\ x))) \text{ of type } \iota \Rightarrow \iota.$

Definition 16 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a)\ P)))$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)})$$
(5)

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal$$
(6)

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal})$$
(7)

Definition 17 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal. (ap\ (c_2Emin_2E_40\ V0a)\ a)$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)})$$
(8)

Definition 18 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal. (ap\ (c_2Emin_2E_40\ V0T1)\ V1T2)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega$$
(9)

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum$$
(10)

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega})$$
(11)

Definition 19 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (12)$$

Definition 20 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (c_2Ebool_2E2$

Definition 21 We define $c_2Ereal_topology_2Elimit_point_of$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1s \in ($

Definition 22 We define $c_2Ereal_topology_2Eclosure$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (ap\ (c_2Epred$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (13)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (14)$$

Definition 23 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 24 We define $c_2Eprim_rec_2E3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 25 We define $c_2Earithmetic_2E3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 26 We define $c_2Earithmetic_2E3E3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Ereal_topology_2Enet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ereal_topology_2Enet\ A0) \quad (15)$$

Let $c_2Ereal_topology_2Emk_net : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow c_2Ereal_topology_2Emk_net \\ A.27a \in ((ty_2Ereal_topology_2Enet\ A.27a)^{(2^{A.27a})^{A.27a}}) \end{aligned} \quad (16)$$

Definition 27 We define $c_2Ereal_topology_2Esequentially$ to be $(ap\ (c_2Ereal_topology_2Emk_net\ ty_2E$

Definition 28 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A.27a : \iota.(\lambda V0x \in A.27a.c_2Ebool_2EF)$.

Definition 29 We define $c_2Epred_set_2EINSERT$ to be $\lambda A.27a : \iota.\lambda V0x \in A.27a.\lambda V1s \in (2^{A.27a}).(ap\ (c_2$

Definition 30 We define $c_2Epred_set_2EDIFF$ to be $\lambda A.27a : \iota.\lambda V0s \in (2^{A.27a}).\lambda V1t \in (2^{A.27a}).(ap\ (c_2$

Definition 31 We define $c_2Epred_set_2EDELETE$ to be $\lambda A.27a : \iota.\lambda V0s \in (2^{A.27a}).\lambda V1x \in A.27a.(ap\ (a$

Let $c_2Ereal_topology_2Enetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Ereal_topology_2Enetord\ A.27a \in ((2^{A.27a})^{A.27a})^{(ty_2Ereal_topology_2Enet\ A.27a)} \quad (17)$$

Definition 32 We define $c_Ereal_topology_Etrivial_limit$ to be $\lambda A_27a : \iota.\lambda V0net \in (ty_2Ereal_topology$

Definition 33 We define $c_Ereal_topology_Eeventually$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1net \in (ty_2$

Definition 34 We define $c_Ereal_topology_E_2D_2D_3E$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal^A$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \vee \neg(p V0t)))) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in A_27b.((\lambda p (\lambda V2x \in A_27b. V0t1) V1t2) = V0t1))) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow \neg(p V0t))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(\neg(p V0t) \Rightarrow ((p V0t) \Rightarrow False))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (28)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (29)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (30)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (32)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\forall V1x \in A.27a.(p(ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p(ap V0P V2x)))))) \quad (33)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\exists V1x \in A.27a.(p(ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p(ap V0P V2x)))))) \quad (34)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in (2^{A.27a}).((\forall V2x \in A.27a.((p(ap V0P V2x)) \wedge (p(ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A.27a.(p(ap V0P V3x))) \wedge (\forall V4x \in A.27a.(p(ap V1Q V4x)))))) \quad (35)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in (2^{A.27a}).((\exists V2x \in A.27a.((p(ap V0P V2x)) \vee (p(ap V1Q V2x)))) \Leftrightarrow ((\exists V3x \in A.27a.(p(ap V0P V3x))) \vee (\exists V4x \in A.27a.(p(ap V1Q V4x)))))) \quad (36)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ 2.((\exists V2x \in A.27a.(p \ (ap \ V0P \ V2x))) \vee (p \ V1Q))) \Leftrightarrow (\exists V3x \in \\ A.27a.((p \ (ap \ V0P \ V3x)) \vee (p \ V1Q)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ 2.((\exists V2x \in A.27a.((p \ (ap \ V0P \ V2x)) \wedge (p \ V1Q))) \Leftrightarrow ((\exists V3x \in \\ A.27a.(p \ (ap \ V0P \ V3x)) \wedge (p \ V1Q)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\ 2^{A.27a}).((\exists V2x \in A.27a.((p \ V0P) \wedge (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((p \\ V0P) \wedge (\exists V3x \in A.27a.(p \ (ap \ V1Q \ V3x)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee (\\ (p \ V1B) \vee (p \ V2C))) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((p \ V0A) \vee (p \ V1B)) \Leftrightarrow ((p \ V1B) \vee \\ (p \ V0A)))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg(\\ p \ V0A)) \vee (\neg(p \ V1B)))))) \wedge (((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \wedge (\neg(p \ V1B)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1a \in \\ A.27a.((\exists V2x \in A.27a.((V2x = V1a) \wedge (p \ (ap \ V0P \ V2x)))) \Leftrightarrow (p \ (\\ ap \ V0P \ V1a)))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow (\\ \forall V0P \in ((2^{A.27b})^{A.27a}).((\forall V1x \in A.27a.(\exists V2y \in \\ A.27b.(p \ (ap \ (ap \ V0P \ V1x) \ V2y)))) \Leftrightarrow (\exists V3f \in (A.27b^{A.27a}).(\\ \forall V4x \in A.27a.(p \ (ap \ (ap \ V0P \ V4x) \ (ap \ V3f \ V4x)))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.((ap \ (c.2Ecombin.2EI \\ A.27a) \ V0x) = V0x)) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\ A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b))) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}). (\forall V1v \in \\ A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\ A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap\ (ap\ (c_2Epair_2E_2C \\ A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ (2^{A_27a}). (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\ V2x)\ (ap\ (ap\ (c_2Epred_set_2EUNION\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ A_27a)\ V2x)\ V1t)))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1x \in \\ A_27a. (\forall V2y \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1x) \\ (ap\ (ap\ (c_2Epred_set_2EDELETE\ A_27a)\ V0s)\ V2y))) \Leftrightarrow ((p\ (ap\ (ap \\ (c_2Ebool_2EIN\ A_27a)\ V1x)\ V0s)) \wedge \neg(V1x = V2y)))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in ty_2Erealx_2Ereal. (\forall V1s \in (2^{ty_2Erealx_2Ereal}). \\ ((p\ (ap\ (ap\ c_2Ereal_topology_2Elimit_point_of\ V0x)\ V1s)) \Leftrightarrow \\ (\exists V2f \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}). ((\forall V3n \in \\ ty_2Enum_2Enum. (p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealx_2Ereal) \\ (ap\ V2f\ V3n))\ (ap\ (ap\ (c_2Epred_set_2EDELETE\ ty_2Erealx_2Ereal) \\ V1s)\ V0x)))) \wedge (p\ (ap\ (ap\ (ap\ (c_2Ereal_topology_2E_2D_2D_3E\ ty_2Enum_2Enum) \\ V2f)\ V0x)\ c_2Ereal_topology_2Esequentially)))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0net \in (ty_2Ereal_topology_2Enet \\ A_27a). (\forall V1a \in ty_2Erealx_2Ereal. (p\ (ap\ (ap\ (ap\ (c_2Ereal_topology_2E_2D_2D_3E \\ A_27a)\ (\lambda V2x \in A_27a.V1a))\ V1a)\ V0net)))) \end{aligned} \quad (51)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (52)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (53)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (54)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (55)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (57)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (58)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (59)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (60)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (62)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (63)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (64)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (65)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (66)$$

Theorem 1

$$\begin{aligned} & (\forall V0s \in (2^{ty_2Erealax_2Ereal}). (\forall V1l \in ty_2Erealax_2Ereal. \\ & ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V1l) (ap c_2Ereal_topology_2Eclosure \\ & V0s))) \Leftrightarrow (\exists V2x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). (\\ & (\forall V3n \in ty_2Enum_2Enum. (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\ & (ap V2x V3n)) V0s))) \wedge (p (ap (ap (ap (c_2Ereal_topology_2E_2D_2D_3E \\ & ty_2Enum_2Enum) V2x) V1l) c_2Ereal_topology_2Esequentially))))))) \end{aligned}$$