thm_2Ereal__topology_2ECOLLINEAR__DIST__IN__CLOSED__SEG (TMMFw-bKw7VZbKcHaNtGNayHMDxWKXrRHBiv)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $c\_2Ereal\_\_topology\_2EDist : \iota$ be given. Assume the following.

$$c\_2Ereal\_\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \tag{3}$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{4}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{5}$$

Definition 7 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 8 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(ty$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{6}$$

Definition 9 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2$

Definition 10 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 11 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 12 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 13 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 14 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{7}$$

Definition 15 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{8}$$

Definition 16 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \tag{9}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{10}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr} \tag{11}$$

Definition 17 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty$

Definition 18 *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{12}$$

Definition 19 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Eh} \tag{13}$$

Definition 20 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Definition 21 *We define* c_2Ereal_2Ereal__sub *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2E$

Definition 22 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 23 *We define* c_2Ereal__topology_2Ecollinear *to be* $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap$ (c_2Ebool_2E

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{14}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{15}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{16}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{17}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{18}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{19}$$

Definition 24 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 25 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0*.*

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{20}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{21}$$

Definition 26 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{22}$$

Definition 27 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 28 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{23}$$

Let $c\_2Elist\_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EHD\ A\_27a \in (A\_27a^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{24}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{25}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{26}$$

Definition 29 *We define* c_2Ereal__topology_2Eclosed__segment *to be* $\lambda V0l \in (ty\_2Elist\_2Elist\ (ty\_2Epair$

Let $c\_2Ereal\_\_topology\_2Ebetween : \iota$ be given. Assume the following.

$$c\_2Ereal\_\_topology\_2Ebetween \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)})^{ty\_2Erealax\_2Erea} \tag{27}$$

Assume the following.

$$True \tag{28}$$

Assume the following.

$$\forall A\_27a.\mathit{nonempty}\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{29}$$

Assume the following.

$$\forall A\_27a.\mathit{nonempty}\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{30}$$

Assume the following.

$$(\forall V0t \in 2.(((\mathit{True} \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow \mathit{True}) \Leftrightarrow (p\ V0t)) \wedge (((\mathit{False} \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow \mathit{False}) \Leftrightarrow (\neg (p\ V0t))))))) \tag{31}$$

Assume the following.

$$(\forall V0a \in \mathit{ty\_2Erealax\_2Ereal}.(\forall V1b \in \mathit{ty\_2Erealax\_2Ereal}.(\forall V2x \in \mathit{ty\_2Erealax\_2Ereal}.(((p\ (\mathit{ap}\ \mathsf{c\_2Ereal\_\_topology\_2Ecollinear}\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ \mathit{ty\_2Erealax\_2Ereal})\ V2x)\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ \mathit{ty\_2Erealax\_2Ereal})\ V0a)\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ \mathit{ty\_2Erealax\_2Ereal})\ V1b)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ \mathit{ty\_2Erealax\_2Ereal})))))) \wedge ((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (\mathit{ap}\ \mathit{c\_2Ereal\_\_topology\_2EDist}\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Epair\_2E\_2C}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal})\ V2x)\ V0a)))\ (\mathit{ap}\ \mathit{c\_2Ereal\_\_topology\_2EDist}\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Epair\_2E\_2C}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal})\ V0a)\ V1b)))) \wedge (p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (\mathit{ap}\ \mathit{c\_2Ereal\_\_topology\_2EDist}\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Epair\_2E\_2C}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal})\ V2x)\ V1b)))\ (\mathit{ap}\ \mathit{c\_2Ereal\_\_topology\_2EDist}\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Epair\_2E\_2C}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal})\ V0a)\ V1b)))))) \Rightarrow (p\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Ereal\_\_topology\_2Ebetween}\ V2x)\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Epair\_2E\_2C}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal})\ V0a)\ V1b))))))) \tag{32}$$

Assume the following.

$$(\forall V0x \in \mathit{ty\_2Erealax\_2Ereal}.(\forall V1a \in \mathit{ty\_2Erealax\_2Ereal}.(\forall V2b \in \mathit{ty\_2Erealax\_2Ereal}.((p\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Ereal\_\_topology\_2Ebetween}\ V0x)\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Epair\_2E\_2C}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal})\ V1a)\ V2b))) \Leftrightarrow (p\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Ebool\_2EIN}\ \mathit{ty\_2Erealax\_2Ereal})\ V0x)\ (\mathit{ap}\ \mathsf{c\_2Ereal\_\_topology\_2Eclosed\_\_segment}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2Elist\_2ECONS}\ (\mathit{ty\_2Epair\_2Eprod}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal}))\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Epair\_2E\_2C}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal})\ V1a)\ V2b))\ (\mathit{c\_2Elist\_2ENIL}\ (\mathit{ty\_2Epair\_2Eprod}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal}))))))))) \tag{33}$$

Theorem 1

$$(\forall V0a \in ty\_2Erealax\_2Ereal.(\forall V1b \in ty\_2Erealax\_2Ereal.$$
$$(\forall V2x \in ty\_2Erealax\_2Ereal.(((p\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Ecollinear}$$
$$(ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ ty\_2Erealax\_2Ereal)\ V2x)\ (ap$$
$$(ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ ty\_2Erealax\_2Ereal)\ V0a)\ (ap\ (ap$$
$$(\mathsf{c\_2Epred\_\_set\_2EINSERT}\ ty\_2Erealax\_2Ereal)\ V1b)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}$$
$$ty\_2Erealax\_2Ereal))))))\wedge((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ ($$
$$ap\ c\_2Ereal\_\_topology\_2EDist\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal$$
$$ty\_2Erealax\_2Ereal)\ V2x)\ V0a)))\ (ap\ c\_2Ereal\_\_topology\_2EDist$$
$$(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)$$
$$V0a)\ V1b))))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_\_topology\_2EDist$$
$$(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)$$
$$V2x)\ V1b)))\ (ap\ c\_2Ereal\_\_topology\_2EDist\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}$$
$$ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V0a)\ V1b))))))\Rightarrow(p\ (ap$$
$$(ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V2x)\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Eclosed\_\_segment}$$
$$(ap\ (ap\ (c\_2Elist\_2ECONS\ (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal$$
$$ty\_2Erealax\_2Ereal))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal$$
$$ty\_2Erealax\_2Ereal)\ V0a)\ V1b))\ (c\_2Elist\_2ENIL\ (ty\_2Epair\_2Eprod$$
$$ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal))))))))))$$