

thm_2Ereal__topology_2Ecompact__bigunion (TMUnnu9eSyCbywQ7zFU4Y2NAzoVg9hMebhj)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2E$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2E$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2E$

Definition 7 We define $c_2Ebool_2E_2EIN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{9}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \tag{10}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{11}$$

Definition 21 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \tag{12}$$

Definition 22 We define $c_2Ereal_2Ereal_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (c_2Ebool_2E2$

Definition 23 We define $c_2Ereal_2Ereal_2EClosed$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ c_2Ereal_2Ereal_2EOpen$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealm_neg \in & ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ & ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \end{aligned} \tag{13}$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \tag{14}$$

Let $c_2Erealax_2Ereal_2EABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_2EABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})}) \tag{15}$$

Definition 24 We define $c_2Erealax_2Ereal_2EABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 25 We define $c_2Erealax_2Ereal_2Eneg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_2EABS$

Definition 26 We define $c_2Ereal_2Ereal_2Elte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 27 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 28 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND$

Definition 29 We define $c_2Ereal_2Ereal_2Ebounded_def$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (c_2Ebool_2ECOND$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{16}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{17}$$

Definition 30 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 31 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 32 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 33 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Ereal_topology_2Enet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ereal_topology_2Enet\ A0) \quad (18)$$

Let $c_2Ereal_topology_2Emk_net : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} &\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ereal_topology_2Emk_net \\ &A_27a \in ((ty_2Ereal_topology_2Enet\ A_27a)^{(2^{A_27a})^{A_27a}}) \end{aligned} \quad (19)$$

Definition 34 We define $c_2Ereal_topology_2Esequentially$ to be $(ap\ (c_2Ereal_topology_2Emk_net\ ty_2Ereal_topology_2Emk_net)$

Definition 35 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g \in (A_27c^{A_27b})$

Let $c_2Ereal_topology_2Enetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ereal_topology_2Enetord\ A_27a \in ((2^{A_27a})^{A_27a})^{(ty_2Ereal_topology_2Enet\ A_27a)} \quad (20)$$

Definition 36 We define $c_2Ereal_topology_2Etrivial_limit$ to be $\lambda A_27a : \iota.\lambda V0net \in (ty_2Ereal_topology_2Emk_net)^A$

Definition 37 We define $c_2Ereal_topology_2Eeventually$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1net \in (ty_2Ereal_topology_2Emk_net)^A$

Definition 38 We define $c_2Ereal_topology_2E_2D_2D_3E$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal)^A$

Definition 39 We define $c_2Ereal_topology_2Ecompact$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (c_2Ebool_2Ebool_2Ebool\ ty_2Erealax_2Ereal)$

Assume the following.

$$True \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (22)$$

Assume the following.

$$\begin{aligned} &(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ &(p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ &(((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} &(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ &True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ &(p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (26)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (27)$$

Assume the following.

$$2. (((((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (28)$$

Assume the following.

$$(\forall V0s \in (2^{(2^{ty_2Erealax_2Ereal})}). (((p\ (ap\ (c_2Epred_set_2EFINITE\ (2^{ty_2Erealax_2Ereal})\ V0s)) \wedge (\forall V1t \in (2^{ty_2Erealax_2Ereal}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{ty_2Erealax_2Ereal})\ V1t)\ V0s)) \Rightarrow (p\ (ap\ c_2Ereal_topology_2EClosed\ V1t)))))) \Rightarrow (p\ (ap\ c_2Ereal_topology_2EClosed\ (ap\ (c_2Epred_set_2EBIGUNION\ ty_2Erealax_2Ereal)\ V0s)))))) \quad (29)$$

Assume the following.

$$(\forall V0f \in (2^{(2^{ty_2Erealax_2Ereal})}). (((p\ (ap\ (c_2Epred_set_2EFINITE\ (2^{ty_2Erealax_2Ereal})\ V0f)) \wedge (\forall V1s \in (2^{ty_2Erealax_2Ereal}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{ty_2Erealax_2Ereal})\ V1s)\ V0f)) \Rightarrow (p\ (ap\ c_2Ereal_topology_2Ebounded_def\ V1s)))))) \Rightarrow (p\ (ap\ c_2Ereal_topology_2Ebounded_def\ (ap\ (c_2Epred_set_2EBIGUNION\ ty_2Erealax_2Ereal)\ V0f)))))) \quad (30)$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}). ((p\ (ap\ c_2Ereal_topology_2Ecompact\ V0s)) \Leftrightarrow ((p\ (ap\ c_2Ereal_topology_2Ebounded_def\ V0s)) \wedge (p\ (ap\ c_2Ereal_topology_2EClosed\ V0s)))))) \quad (31)$$

Theorem 1

$$(\forall V0s \in (2^{(2^{ty_2Erealax_2Ereal})}). (((p\ (ap\ (c_2Epred_set_2EFINITE\ (2^{ty_2Erealax_2Ereal})\ V0s)) \wedge (\forall V1t \in (2^{ty_2Erealax_2Ereal}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{ty_2Erealax_2Ereal})\ V1t)\ V0s)) \Rightarrow (p\ (ap\ c_2Ereal_topology_2Ecompact\ V1t)))))) \Rightarrow (p\ (ap\ c_2Ereal_topology_2Ecompact\ (ap\ (c_2Epred_set_2EBIGUNION\ ty_2Erealax_2Ereal)\ V0s))))))$$