

# thm\_2Ereal\_\_topology\_2ECOMPACT\_\_EQ\_\_BOUNDED\_\_CLOSED (TMLCq7KQgTh3yJgbSZvjmw2Gfz5PEMZRP55)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \tag{4}$$

**Definition 3** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$  then (the  $(\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ ).

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 5** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E\_40 (ty$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal}) \tag{5}$$

Let  $c\_2Erealax\_2Etreal\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)) \quad (6)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (7)$$

**Definition 6** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 7** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_neg)$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in omega \quad (8)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (9)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum)^{omega} \quad (10)$$

**Definition 8** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal)^{ty\_2Enum\_2Enum} \quad (11)$$

Let  $c\_2Erealax\_2Etreal\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)) \quad (12)$$

**Definition 9** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 10** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 11** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 12** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_21))$

**Definition 13** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 14** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t))))$

**Definition 15** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap\ c\_2Ebool\_2E\_21\ 2\ V2t2))))$

**Definition 16** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap (ap (ap (c\_2Ebool\_2ECONI$

**Definition 17** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x))$

**Definition 18** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40$

**Definition 19** We define  $c\_2Ereal\_topology\_2Ebounded\_def$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap (c\_2Ebo$

**Definition 20** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E2ET).$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (13)$$

**Definition 21** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b})}) \end{aligned} \quad (14)$$

**Definition 22** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2$

Let  $c\_2Ereal\_topology\_2EDist : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal)}) \quad (15)$$

**Definition 23** We define  $c\_2Ereal\_topology\_2EOpen$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap (c\_2Ebool\_2E\_2$

**Definition 24** We define  $c\_2Ereal\_topology\_2EClosed$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap c\_2Ereal\_topo$

**Definition 25** We define  $c\_2Ereal\_topology\_2Elimit\_point\_of$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1s \in ($

**Definition 26** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap ($

**Definition 27** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in$

**Definition 28** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap (c\_2$

**Definition 29** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E2EF).$

**Definition 30** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_21 (2$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (16)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (17)$$

**Definition 31** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$

**Definition 32** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 33** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 34** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $ty\_2Ereal\_topology\_2Enet : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ereal\_topology\_2Enet\ A0) \quad (18)$$

Let  $c\_2Ereal\_topology\_2Emk\_net : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow c\_2Ereal\_topology\_2Emk\_net \\ A.27a \in ((ty\_2Ereal\_topology\_2Enet\ A.27a)^{(2^{A.27a})^{A.27a}}) \end{aligned} \quad (19)$$

**Definition 35** We define  $c\_2Ereal\_topology\_2Esequentially$  to be  $(ap\ (c\_2Ereal\_topology\_2Emk\_net\ ty\_2Ereal\_topology\_2Enet$

**Definition 36** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.\lambda V0f \in (A.27b^{A.27c}).\lambda V1g$

Let  $c\_2Ereal\_topology\_2Enetord : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c\_2Ereal\_topology\_2Enetord\ A.27a \in \left( (2^{A.27a})^{A.27a} (ty\_2Ereal\_topology\_2Enet\ A.27a) \right) \quad (20)$$

**Definition 37** We define  $c\_2Ereal\_topology\_2Etrivial\_limit$  to be  $\lambda A.27a : \iota.\lambda V0net \in (ty\_2Ereal\_topology\_2Enet$

**Definition 38** We define  $c\_2Ereal\_topology\_2Eeventually$  to be  $\lambda A.27a : \iota.\lambda V0p \in (2^{A.27a}).\lambda V1net \in (ty\_2Ereal\_topology\_2Enet$

**Definition 39** We define  $c\_2Ereal\_topology\_2E\_2D\_2D\_3E$  to be  $\lambda A.27a : \iota.\lambda V0f \in (ty\_2Erealax\_2Ereal^{A.27a}$

**Definition 40** We define  $c\_2Ereal\_topology\_2Ecompact$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap\ (c\_2Ebool\_2Ereal$

Assume the following.

$$True \quad (21)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge \\ ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \end{aligned} \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow \neg(p\ V0t))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\ & A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ & p V0t)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\forall V1x \in \\ & A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\exists V1x \in \\ & A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ & 2.(((\forall V2x \in A.27a.(p (ap V0P V2x))) \wedge (p V1Q)) \Leftrightarrow (\forall V3x \in \\ & A.27a.((p (ap V0P V3x)) \wedge (p V1Q)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ( \\ & 2^{A.27a}).(((p V0P) \wedge (\forall V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in \\ & A.27a.((p V0P) \wedge (p (ap V1Q V3x)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ & 2.(((\exists V2x \in A.27a.(p (ap V0P V2x))) \vee (p V1Q)) \Leftrightarrow (\exists V3x \in \\ & A.27a.((p (ap V0P V3x)) \vee (p V1Q)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((p V0P) \vee (\exists V2x \in A.27a. (p (ap V1Q V2x)))))) \Leftrightarrow (\exists V3x \in A.27a. ((p V0P) \vee (p (ap V1Q V3x)))))) \quad (35)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in 2. ((\exists V2x \in A.27a. ((p (ap V0P V2x)) \wedge (p V1Q)))) \Leftrightarrow ((\exists V3x \in A.27a. (p (ap V0P V3x))) \wedge (p V1Q)))))) \quad (36)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((\exists V2x \in A.27a. ((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \wedge (\exists V3x \in A.27a. (p (ap V1Q V3x)))))) \quad (37)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p (ap V1P V2x)) \vee (p V0Q)))) \Leftrightarrow ((\forall V3x \in A.27a. (p (ap V1P V3x))) \vee (p V0Q)))) \quad (38)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a. (p (ap V1Q V3x)))))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B)) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B)))))) \quad (42)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\forall V0P \in ((2^{A.27b})^{A.27a}). ((\forall V1x \in A.27a. (\exists V2y \in A.27b. (p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A.27b^{A.27a}). (\forall V4x \in A.27a. (p (ap (ap V0P V4x) (ap V3f V4x)))))) \quad (43)$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(((p (ap c\_2Ereal\_topology\_2Ebunded\_def V0s)) \wedge (p (ap c\_2Ereal\_topology\_2EClosed V0s))) \Rightarrow (p (ap c\_2Ereal\_topology\_2Ecompact V0s)))) \quad (44)$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).((\forall V1t \in (2^{ty\_2Erealax\_2Ereal}).(((\neg(p (ap (c\_2Epred\_set\_2EFINITE ty\_2Erealax\_2Ereal) V1t))) \wedge (p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) V1t) V0s)))) \Rightarrow (\exists V2x \in ty\_2Erealax\_2Ereal.(p (ap (ap c\_2Ereal\_topology\_2Elimit\_point\_of V2x) V1t)))))) \Rightarrow (p (ap c\_2Ereal\_topology\_2Ebunded\_def V0s)))) \quad (45)$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).((\forall V1t \in (2^{ty\_2Erealax\_2Ereal}).(((\neg(p (ap (c\_2Epred\_set\_2EFINITE ty\_2Erealax\_2Ereal) V1t))) \wedge (p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) V1t) V0s)))) \Rightarrow (\exists V2x \in ty\_2Erealax\_2Ereal.((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V2x) V0s)) \wedge (p (ap (ap c\_2Ereal\_topology\_2Elimit\_point\_of V2x) V1t)))))) \Rightarrow (p (ap c\_2Ereal\_topology\_2EClosed V0s)))))) \quad (46)$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).((p (ap c\_2Ereal\_topology\_2Ecompact V0s)) \Leftrightarrow (\forall V1t \in (2^{ty\_2Erealax\_2Ereal}).(((\neg(p (ap (c\_2Epred\_set\_2EFINITE ty\_2Erealax\_2Ereal) V1t))) \wedge (p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) V1t) V0s)))) \Rightarrow (\exists V2x \in ty\_2Erealax\_2Ereal.((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V2x) V0s)) \wedge (p (ap (ap c\_2Ereal\_topology\_2Elimit\_point\_of V2x) V1t)))))))))) \quad (47)$$

Assume the following.

$$(\forall V0t \in 2.(((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (48)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (49)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (50)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (51)$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (52)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg( \\ & p V2r)) \vee (\neg(p V1q)))))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (( \\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (57)$$

### Theorem 1

$$\begin{aligned} & (\forall V0s \in (2^{ty-2Erealax-2Ereal}).((p (ap c.2Ereal\_topology\_2Ecompact \\ & V0s)) \Leftrightarrow ((p (ap c.2Ereal\_topology\_2Ebounded\_def V0s)) \wedge (p (ap \\ & c.2Ereal\_topology\_2EClosed V0s)))))) \end{aligned}$$