

thm_2Ereal__topology_2ECOMPACT__FRONTIER__BOUNDED (TMPGJCu6P8toidLyGkjMHN6mDuhnan2hxc5)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p (ap P x))$) of type $\iota \Rightarrow \iota$.

Definition 4 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a P))))$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Definition 5 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}) P) (c_2Emin_2E_3D (2^{A_27a}) P))))$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \tag{3}$$

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (4)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (5)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (6)$$

Definition 10 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal. (ap (c_2Emin_2E_40 (t$

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (7)$$

Definition 11 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (8)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (10)$$

Definition 12 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (11)$$

Definition 13 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}). (ap (c_2Ebool_2E_2$

Definition 14 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap ($

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (12)$$

Definition 15 We define $c_Ereal_topology_Einterior$ to be $\lambda V0s \in (2^{ty_Erealax_Ereal}).(ap (c_Epred_s$

Definition 16 We define c_Ebool_E2E to be $(ap (c_Ebool_E2E_21) 2) (\lambda V0t \in 2.V0t)$.

Definition 17 We define $c_Ebool_E2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_E3D_3D_3E V0t) c_Ebool_E2E$

Definition 18 We define $c_Epred_set_E2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2$

Definition 19 We define $c_Ereal_topology_Elimit_point_of$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1s \in ($

Definition 20 We define $c_Ebool_E2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E2E_21) 2) (\lambda V2t \in$

Definition 21 We define $c_Epred_set_E2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c$

Definition 22 We define $c_Ereal_topology_E2Eclosure$ to be $\lambda V0s \in (2^{ty_Erealax_Ereal}).(ap (ap (c_Epred$

Definition 23 We define $c_Ereal_topology_E2Efrontier$ to be $\lambda V0s \in (2^{ty_Erealax_Ereal}).(ap (ap (c_Epred$

Definition 24 We define $c_Epred_set_E2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_Ebool_E2E)$.

Definition 25 We define $c_Ereal_topology_E2EClosed$ to be $\lambda V0s \in (2^{ty_Erealax_Ereal}).(ap c_Ereal_topo$

Let $c_Erealax_E2Etrealm_neg : \iota$ be given. Assume the following.

$$\begin{aligned} c_Erealax_E2Etrealm_neg \in & ((ty_E2Epair_E2Eprod ty_E2Ehreal_E2Ehreal \\ & ty_E2Ehreal_E2Ehreal)^{(ty_E2Epair_E2Eprod ty_E2Ehreal_E2Ehreal ty_E2Ehreal_E2Ehreal)}) \end{aligned} \quad (13)$$

Let $c_Erealax_E2Etrealm_eq : \iota$ be given. Assume the following.

$$c_Erealax_E2Etrealm_eq \in ((2^{(ty_E2Epair_E2Eprod ty_E2Ehreal_E2Ehreal ty_E2Ehreal_E2Ehreal)})^{(ty_E2Epair_E2Eprod ty_E2Ehreal_E2Ehreal ty_E2Ehreal_E2Ehreal)}) \quad (14)$$

Let $c_Erealax_E2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_E2Ereal_ABS_CLASS \in (ty_E2Erealax_E2Ereal)^{(2^{(ty_E2Epair_E2Eprod ty_E2Ehreal_E2Ehreal ty_E2Ehreal_E2Ehreal)})} \quad (15)$$

Definition 26 We define $c_Erealax_E2Ereal_ABS$ to be $\lambda V0r \in (ty_E2Epair_E2Eprod ty_E2Ehreal_E2Ehreal ty$

Definition 27 We define $c_Erealax_E2Ereal_neg$ to be $\lambda V0T1 \in ty_E2Erealax_E2Ereal.(ap c_Erealax_E2Ereal$

Definition 28 We define $c_Ereal_E2Ereal_lte$ to be $\lambda V0x \in ty_E2Erealax_E2Ereal.\lambda V1y \in ty_E2Erealax_E2Ereal$

Definition 29 We define $c_Ebool_E2ECOND$ to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 30 We define c_Ereal_E2Eabs to be $\lambda V0x \in ty_E2Erealax_E2Ereal.(ap (ap (ap (c_Ebool_E2ECON$

Definition 31 We define $c_Ereal_topology_E2Ebounded_def$ to be $\lambda V0s \in (2^{ty_Erealax_E2Ereal}).(ap (c_Ebo$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\ & A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\forall V1x \in \\ & A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (\\ & (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee \\ & (p V0A)))) \end{aligned} \quad (32)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))))) \quad (33)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (34)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_{27} \in 2. (\forall V2y \in 2. (\forall V3y_{27} \in 2. (((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \quad (35)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0s \in (2^{A_{27a}}). (\forall V1t \in (2^{A_{27a}}). (p (ap (ap (c_{2Epred_set_2ESUBSET } A_{27a}) (ap (ap (c_{2Epred_set_2EDIFF } A_{27a}) V0s) V1t)) V0s)))) \quad (36)$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealx_2Ereal}). (\forall V1t \in (2^{ty_2Erealx_2Ereal}). (((p (ap c_{2Ereal_topology_2EClosed } V0s)) \wedge (p (ap c_{2Ereal_topology_2EOpen } V1t))) \Rightarrow (p (ap c_{2Ereal_topology_2EClosed } (ap (ap (c_{2Epred_set_2EDIFF } ty_2Erealx_2Ereal) V0s) V1t)))))) \quad (37)$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealx_2Ereal}). (p (ap c_{2Ereal_topology_2EOpen } (ap c_{2Ereal_topology_2Einterior } V0s)))) \quad (38)$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealx_2Ereal}). (p (ap c_{2Ereal_topology_2EClosed } (ap c_{2Ereal_topology_2Eclosure } V0s)))) \quad (39)$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealx_2Ereal}). (\forall V1t \in (2^{ty_2Erealx_2Ereal}). (((p (ap c_{2Ereal_topology_2Ebounded_def } V1t)) \wedge (p (ap (ap (c_{2Epred_set_2ESUBSET } ty_2Erealx_2Ereal) V0s) V1t))) \Rightarrow (p (ap c_{2Ereal_topology_2Ebounded_def } V0s)))))) \quad (40)$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealx_2Ereal}). (((p (ap c_{2Ereal_topology_2Ebounded_def } V0s)) \Rightarrow (p (ap c_{2Ereal_topology_2Ebounded_def } (ap c_{2Ereal_topology_2Eclosure } V0s)))))) \quad (41)$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}).((p (ap\ c_2Ereal_topology_2Ecompact\ V0s)) \Leftrightarrow ((p (ap\ c_2Ereal_topology_2Ebunded_def\ V0s)) \wedge (p (ap\ c_2Ereal_topology_2EClosed\ V0s)))))) \quad (42)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (44)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (45)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (46)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow ((p\ V0A) \Rightarrow False))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((\neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \quad (51)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (52)$$

Theorem 1

$$(\forall V0s \in (2^{ty-2Erealax-2Ereal}). ((p (ap c_2Ereal_topology_2Ebounded_def V0s)) \Rightarrow (p (ap c_2Ereal_topology_2Ecompact (ap c_2Ereal_topology_2Efrontier V0s))))))$$