

thm_2Ereal__topology_2Ecompact__imp__bounded
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cYi7TXFGsFQCwQ6uY188M9wXmY6k8XEK)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Definition 7 We define $c_2Epred_set_2Euniv$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E_2T)$.

Definition 8 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Epair_2Eabs_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2Eabs_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \tag{3}$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epred_set_2EGSPEC \\ A.27a A.27b \in ((2^{A.27a})^{(ty_2Epair_2Eprod A.27a 2)^{A.27b}})$$
(4)

Definition 11 We define $c_2Epred_set_2EDIFF$ to be $\lambda A.27a : \iota.\lambda V0s \in (2^{A.27a}).\lambda V1t \in (2^{A.27a}).(ap (c_2Ereal_topology_2EDist : \iota)$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)})$$
(5)

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty ty_2Ehreal_2Ehreal$$
(6)

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal})$$
(7)

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$.

Definition 13 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (t$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)})$$
(8)

Definition 14 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega$$
(9)

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum$$
(10)

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega})$$
(11)

Definition 15 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum})$$
(12)

Definition 16 We define c_Ebool_E3F to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_Emin_E40$

Definition 17 We define $c_Ereal_topology_EOpen$ to be $\lambda V0s \in (2^{ty_Erealax_Ereal}). (ap (c_Ebool_E2$

Definition 18 We define $c_Ereal_topology_EClosed$ to be $\lambda V0s \in (2^{ty_Erealax_Ereal}). (ap c_Ereal_topo$

Let $c_Erealax_Etrealm_neg : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_neg \in ((ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal) (ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)) \quad (13)$$

Let $c_Erealax_Etrealm_eq : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_eq \in ((2^{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)}) (ty_Epair_Eprod ty_Ehreal_Ehreal)) \quad (14)$$

Let $c_Erealax_Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_Ereal_ABS_CLASS \in (ty_Erealax_Ereal)^{(2^{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)})} \quad (15)$$

Definition 19 We define $c_Erealax_Ereal_ABS$ to be $\lambda V0r \in (ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)$

Definition 20 We define $c_Erealax_Ereal_neg$ to be $\lambda V0T1 \in ty_Erealax_Ereal. (ap c_Erealax_Ereal.$

Definition 21 We define $c_Ereal_Ereal_lte$ to be $\lambda V0x \in ty_Erealax_Ereal. \lambda V1y \in ty_Erealax_Ereal.$

Definition 22 We define c_Ebool_ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 23 We define c_Ereal_Eabs to be $\lambda V0x \in ty_Erealax_Ereal. (ap (ap (ap (c_Ebool_ECOND$

Definition 24 We define $c_Ereal_topology_Ebounded_def$ to be $\lambda V0s \in (2^{ty_Erealax_Ereal}). (ap (c_Ebo$

Let $c_Eenum_EREP_num : \iota$ be given. Assume the following.

$$c_Eenum_EREP_num \in (\omega^{ty_Eenum_Eenum}) \quad (16)$$

Let $c_Eenum_ESUC_REP : \iota$ be given. Assume the following.

$$c_Eenum_ESUC_REP \in (\omega^{\omega}) \quad (17)$$

Definition 25 We define c_Eenum_ESUC to be $\lambda V0m \in ty_Eenum_Eenum. (ap c_Eenum_EABS_num$

Definition 26 We define $c_Eprim_rec_E3C$ to be $\lambda V0m \in ty_Eenum_Eenum. \lambda V1n \in ty_Eenum_Eenum.$

Definition 27 We define $c_Earithmic_E3E$ to be $\lambda V0m \in ty_Eenum_Eenum. \lambda V1n \in ty_Eenum_Eenum.$

Definition 28 We define $c_Ebool_E5C_E2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_Ebool_E21 2) (\lambda V2t$

Definition 29 We define $c_Earithmic_E3E_E3D$ to be $\lambda V0m \in ty_Eenum_Eenum. \lambda V1n \in ty_Eenum_Eenum.$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (26)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_{.27} \in 2. (\forall V2y \in 2. (\forall V3y_{.27} \in 2. (((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (27)$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}). ((p (ap c_2Ereal_topology_2Ecompact V0s)) \Leftrightarrow ((p (ap c_2Ereal_topology_2Ebounded_def V0s)) \wedge (p (ap c_2Ereal_topology_2EClosed V0s))))) \quad (28)$$

Theorem 1

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}). ((p (ap c_2Ereal_topology_2Ecompact V0s)) \Rightarrow (p (ap c_2Ereal_topology_2Ebounded_def V0s))))$$