

thm_2Ereal__topology_2E complement__connected__combi (TMcy1tcWhihkuDuBSRd7BN2K4SeqqaJL1YZ)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 4 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 5 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a})) A_27a))$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) P)))$

Definition 7 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2)) (\lambda V0t \in 2.V0t)$.

Definition 8 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 9 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 10 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2)) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (7)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (8)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (9)$$

Definition 25 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ (t$

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (10)$$

Definition 26 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \quad (11)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (12)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (13)$$

Definition 27 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (14)$$

Definition 28 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (c_2Ebool_2E2$

Definition 29 We define $c_2Ereal_topology_2Econnected$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ c_2Ebool_2E2$

Definition 30 We define $c_2Ereal_topology_2Econnected_component$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).\lambda V$

Definition 31 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A.27a : \iota.\lambda V0P \in (2^{(2^{A-27a})}).(ap\ (c_2Epred_s$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\neg(\forall V1x \in A_27a. (p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). (((p\ V0P) \wedge (\forall V2x \in A_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x))))))) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). (((p\ V0P) \vee (\exists V2x \in A_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\exists V3x \in A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V3x))))))) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in 2. ((\exists V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ V1Q))) \Leftrightarrow ((\exists V3x \in A_27a. (p\ (ap\ V0P\ V3x))) \wedge (p\ V1Q)))))) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in A_27a. (p\ (ap\ V1P\ V3x))) \vee (p\ V0Q)))))) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A_27a. (p\ (ap\ V1Q\ V3x))))))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B))))))))) \quad (37)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (38)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \quad (39)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow (\forall V0P \in ((2^{A_{27b}})^{A_{27a}}).((\forall V1x \in A_{27a}.(\exists V2y \in A_{27b}.(p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A_{27b}^{A_{27a}}).(\forall V4x \in A_{27a}.(p (ap (ap V0P V4x) (ap V3f V4x)))))))))) \quad (40)$$

Assume the following.

$$(\forall V0r \in 2.(\forall V1p \in 2.(\forall V2q \in 2.(((p V1p) \wedge (p V2q)) \Rightarrow (p V0r)) \Leftrightarrow ((p V1p) \Rightarrow ((p V2q) \Rightarrow (p V0r)))))) \quad (41)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0x \in A_{27a}.((ap (c_{2Ecombin_2EI} A_{27a}) V0x) = V0x)) \quad (42)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0s \in (2^{A_{27a}}).(\forall V1t \in (2^{A_{27a}}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_{27a}.((p (ap (ap (c_{2Ebool_2EIN} A_{27a}) V2x) V0s)) \Leftrightarrow (p (ap (ap (c_{2Ebool_2EIN} A_{27a}) V2x) V1t))))))))) \quad (43)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.(\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_{.27a})\ V0x)\ (c_2Epred_set_2EEMPTY\ A_{.27a})))))) \quad (44)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in (2^{A_{.27a}}).(\forall V2x \in A_{.27a}.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_{.27a})\ V2x)\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A_{.27a})\ V0s)\ V1t)))) \Leftrightarrow ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_{.27a})\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_{.27a})\ V2x)\ V1t)))))))))) \quad (45)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in (2^{A_{.27a}}).(\forall V2x \in A_{.27a}.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_{.27a})\ V2x)\ (ap\ (ap\ (c_2Epred_set_2EDIFF\ A_{.27a})\ V0s)\ V1t)))) \Leftrightarrow ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_{.27a})\ V2x)\ V0s)) \wedge (\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_{.27a})\ V2x)\ V1t)))))))))) \quad (46)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1x \in A_{.27a}.(\forall V2y \in A_{.27a}.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_{.27a})\ V1x)\ (ap\ (ap\ (c_2Epred_set_2EDELETE\ A_{.27a})\ V0s)\ V2y)))) \Leftrightarrow ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_{.27a})\ V1x)\ V0s)) \wedge (\neg(V1x = V2y)))))))))) \quad (47)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1sos \in (2^{(2^{A_{.27a}})}).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_{.27a})\ V0x)\ (ap\ (c_2Epred_set_2EBIGUNION\ A_{.27a})\ V1sos)))) \Leftrightarrow (\exists V2s \in (2^{A_{.27a}}).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_{.27a})\ V0x)\ V2s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_{.27a}})\ V2s)\ V1sos)))))))))) \quad (48)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow \forall A.27e.nonempty \\
& A.27e \Rightarrow \forall A.27f.nonempty\ A.27f \Rightarrow \forall A.27g.nonempty\ A.27g \Rightarrow \\
& (\forall V0Q \in (2^{A.27b}).((\forall V1P \in (2^{A.27a}).(\forall V2f \in \\
& (A.27b^{A.27a}).((\forall V3z \in A.27b.((p\ (ap\ (ap\ (c.2Ebool.2EIN \\
& A.27b)\ V3z)\ (ap\ (c.2Epred_set.2EGSPEC\ A.27b\ A.27a)\ (\lambda V4x \in \\
& A.27a.(ap\ (ap\ (c.2Epair.2E.2C\ A.27b\ 2)\ (ap\ V2f\ V4x))\ (ap\ V1P\ V4x)))))) \Rightarrow \\
& (p\ (ap\ V0Q\ V3z)))) \Leftrightarrow (\forall V5x \in A.27a.((p\ (ap\ V1P\ V5x)) \Rightarrow (p\ (ap\ V0Q \\
& (ap\ V2f\ V5x)))))) \wedge ((\forall V6P \in ((2^{A.27d})^{A.27c}).(\forall V7f \in \\
& ((A.27b^{A.27d})^{A.27c}).((\forall V8z \in A.27b.((p\ (ap\ (ap\ (c.2Ebool.2EIN \\
& A.27b)\ V8z)\ (ap\ (c.2Epred_set.2EGSPEC\ A.27b\ (ty.2Epair.2Eprod \\
& A.27c\ A.27d))\ (ap\ (c.2Epair.2EUNCURRY\ A.27c\ A.27d\ (ty.2Epair.2Eprod \\
& A.27b\ 2))\ (\lambda V9x \in A.27c.(\lambda V10y \in A.27d.(ap\ (ap\ (c.2Epair.2E.2C \\
& A.27b\ 2)\ (ap\ (ap\ V7f\ V9x)\ V10y))\ (ap\ (ap\ V6P\ V9x)\ V10y)))))) \Rightarrow (p \\
& (ap\ V0Q\ V8z)))) \Leftrightarrow (\forall V11x \in A.27c.(\forall V12y \in A.27d.((p \\
& (ap\ (ap\ V6P\ V11x)\ V12y)) \Rightarrow (p\ (ap\ V0Q\ (ap\ (ap\ V7f\ V11x)\ V12y)))))) \wedge \\
& (\forall V13P \in (((2^{A.27g})^{A.27f})^{A.27e}).(\forall V14f \in (((A.27b^{A.27g})^{A.27f})^{A.27e}). \\
& (\forall V15z \in A.27b.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27b)\ V15z)\ (\\
& ap\ (c.2Epred_set.2EGSPEC\ A.27b\ (ty.2Epair.2Eprod\ A.27e\ (ty.2Epair.2Eprod \\
& A.27f\ A.27g)))\ (ap\ (c.2Epair.2EUNCURRY\ A.27e\ (ty.2Epair.2Eprod \\
& A.27f\ A.27g)\ (ty.2Epair.2Eprod\ A.27b\ 2))\ (\lambda V16w \in A.27e.(ap \\
& (c.2Epair.2EUNCURRY\ A.27f\ A.27g\ (ty.2Epair.2Eprod\ A.27b\ 2)) \\
& (\lambda V17x \in A.27f.(\lambda V18y \in A.27g.(ap\ (ap\ (c.2Epair.2E.2C\ A.27b \\
& 2)\ (ap\ (ap\ (ap\ V14f\ V16w)\ V17x)\ V18y))\ (ap\ (ap\ (ap\ V13P\ V16w)\ V17x \\
& V18y)))))) \Rightarrow (p\ (ap\ V0Q\ V15z)))) \Leftrightarrow (\forall V19w \in A.27e.(\forall V20x \in \\
& A.27f.(\forall V21y \in A.27g.((p\ (ap\ (ap\ (ap\ V13P\ V19w)\ V20x)\ V21y)) \Rightarrow \\
& (p\ (ap\ V0Q\ (ap\ (ap\ (ap\ V14f\ V19w)\ V20x)\ V21y)))))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty.2Erealx.2Ereal}).(\forall V1x \in ty.2Erealx.2Ereal. \\
& (p\ (ap\ (ap\ (c.2Epred_set.2ESUBSET\ ty.2Erealx.2Ereal)\ (ap\ (ap \\
& c.2Ereal_topology.2Econnected_component\ V0s)\ V1x))\ V0s))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty.2Erealx.2Ereal}).(\forall V1a \in ty.2Erealx.2Ereal. \\
& (\forall V2b \in ty.2Erealx.2Ereal.((p\ (ap\ (ap\ (c.2Epred_set.2EDISJOINT \\
& ty.2Erealx.2Ereal)\ (ap\ (ap\ c.2Ereal_topology.2Econnected_component \\
& V0s)\ V1a))\ (ap\ (ap\ c.2Ereal_topology.2Econnected_component \\
& V0s)\ V2b)))) \Leftrightarrow (\neg (p\ (ap\ (ap\ (c.2Ebool.2EIN\ ty.2Erealx.2Ereal)\ V1a) \\
& (ap\ (ap\ c.2Ereal_topology.2Econnected_component\ V0s)\ V2b))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1x \in ty_2Erealax_2Ereal. \\
& (\forall V2y \in ty_2Erealax_2Ereal.(((ap (ap c_2Ereal_topology_2Econnected_component \\
& V0s) V1x) = (ap (ap c_2Ereal_topology_2Econnected_component \\
& V0s) V2y)) \Leftrightarrow (((\neg(p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V1x) V0s))) \wedge (\neg(p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V2y) \\
& V0s)))) \vee ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V1x) V0s)) \wedge \\
& ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V2y) V0s)) \wedge (p (\\
& ap (ap (ap c_2Ereal_topology_2Econnected_component V0s) V1x) \\
& V2y))))))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).((ap (c_2Epred_set_2EBIGUNION \\
& ty_2Erealax_2Ereal) (ap (c_2Epred_set_2EGSPEC (2^{ty_2Erealax_2Ereal} \\
& ty_2Erealax_2Ereal) (\lambda V1x \in ty_2Erealax_2Ereal.(ap (ap (c_2Epair_2E_2C \\
& (2^{ty_2Erealax_2Ereal} 2) (ap (ap c_2Ereal_topology_2Econnected_component \\
& V0s) V1x)) (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V1x) V0s)))))) = \\
& V0s))
\end{aligned} \tag{53}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{54}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{57}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{59}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \quad (60)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (62)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (63)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (64)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (65)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (66)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (67)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (68)$$

Theorem 1

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}). (\forall V1x \in ty_2Erealax_2Ereal. ((ap (ap (c_2Epred_set_2EDIFF ty_2Erealax_2Ereal) V0s) (ap (ap c_2Ereal_topology_2Econnected_component V0s) V1x)) = (ap (c_2Epred_set_2EBIGUNION ty_2Erealax_2Ereal) (ap (ap (c_2Epred_set_2EDELETE (2^{ty_2Erealax_2Ereal})) (ap (c_2Epred_set_2EGSPEC (2^{ty_2Erealax_2Ereal}) ty_2Erealax_2Ereal) (\lambda V2y \in ty_2Erealax_2Ereal. (ap (ap (c_2Epair_2E_2C (2^{ty_2Erealax_2Ereal}) 2) (ap (ap c_2Ereal_topology_2Econnected_component V0s) V2y)) (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V2y) V0s)))))) (ap (ap c_2Ereal_topology_2Econnected_component V0s) V1x))))))$$