

# thm\_2Ereal\_\_topology\_2ECONNECTED\_\_CLOSED\_\_IN (TMS2Dxmn4Etq16juJUKGWjmNTWWdY6aDvRr)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealx\_2Ereal \tag{1}$$

**Definition 4** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 6** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{3}$$

**Definition 7** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2E$

Let  $c\_2Ereal\_topology\_2EDist : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \quad (4)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (5)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax}) \quad (6)$$

**Definition 8** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A$ . **if**  $(\exists x \in A.p (ap\ P\ x))$  **then** (the  $(\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ ).

**Definition 9** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E\_40\ (ty$

Let  $c\_2Erealax\_2Etreall\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal)}) \quad (7)$$

**Definition 10** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in omega \quad (8)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (9)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (10)$$

**Definition 11** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (11)$$

**Definition 12** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A.27a : \iota.( \lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 13** We define  $c\_2Ereal\_topology\_2EOpen$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap\ (c\_2Ebool\_2E\_2$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (12)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c\_2Etopology\_2Etopology\ A.27a \in ((ty\_2Etopology\_2Etopology\ A.27a)^{(2^{(2^A-27a)}})) \quad (13)$$

**Definition 14** We define  $c\_2Ereal\_topology\_2Eeuclidean$  to be  $(ap (c\_2Etopology\_2Etopology ty\_2Erealax$

Let  $c\_2Etopology\_2Eopen\_in : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Etopology\_2Eopen\_in A\_27a \in ((2^{(2^{A-27a})})^{(ty\_2Etopology\_2Etopology A\_27a)}) \quad (14)$$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A-27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A-27b}}) \quad (15)$$

**Definition 15** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap (c\_2E$

**Definition 16** We define  $c\_2Ereal\_topology\_2Esubtopology$  to be  $\lambda A\_27a : \iota. \lambda V0top \in (ty\_2Etopology\_2Etopology$

**Definition 17** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota. \lambda V0P \in (2^{(2^{A-27a})}). (ap (c\_2Epred\_set$

**Definition 18** We define  $c\_2Etopology\_2Etopspace$  to be  $\lambda A\_27a : \iota. \lambda V0top \in (ty\_2Etopology\_2Etopology$

**Definition 19** We define  $c\_2Ebool\_2E2F$  to be  $(ap (c\_2Ebool\_2E21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 20** We define  $c\_2Ebool\_2E7E$  to be  $(\lambda V0t \in 2. (ap (ap c\_2Emin\_2E3D\_3D\_3E V0t) c\_2Ebool\_2E$

**Definition 21** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap (c\_2E$

**Definition 22** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap ($

**Definition 23** We define  $c\_2Etopology\_2Eclosed\_in$  to be  $\lambda A\_27a : \iota. \lambda V0top \in (ty\_2Etopology\_2Etopology$

**Definition 24** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. c\_2Ebool\_2E2F)$ .

**Definition 25** We define  $c\_2Ebool\_2E5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E21 2) (\lambda V2t \in$

**Definition 26** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap (c\_2E$

**Definition 27** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. c\_2Ebool\_2E2F)$ .

**Definition 28** We define  $c\_2Ereal\_topology\_2EClosed$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}). (ap c\_2Ereal\_topo$

**Definition 29** We define  $c\_2Ereal\_topology\_2Econnected$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}). (ap c\_2Ebool\_2E$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (19)$$

Assume the following.

$$\begin{aligned} &(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ &(p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ &(((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} &(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ &(((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ &(p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} &(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ &True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ &(p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} &((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ &((\neg False) \Leftrightarrow True))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} &\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in A.27a.(\forall V1y \in \\ &A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} &(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ &(p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ &p V0t))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} &\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\neg(\forall V1x \in \\ &A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p (ap V0P V2x))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} &\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ &2.((\exists V2x \in A.27a.(p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in \\ &A.27a.(p (ap V0P V3x)) \wedge (p V1Q)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A\_27a}). ((\exists V2x \in A\_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \wedge (\exists V3x \in A\_27a. (p\ (ap\ V1Q\ V3x))))))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))))) \wedge (((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \quad (29)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (30)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \quad (31)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1a \in A\_27a. ((\exists V2x \in A\_27a. ((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a)))))) \quad (32)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in (2^{A\_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V1t)))))) \quad (33)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\neg(p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ (c\_2Epred\_set\_2EEMPTY\ A\_27a)))))) \quad (34)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in (2^{A\_27a}). (\forall V2x \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ (ap\ (ap\ (c\_2Epred\_set\_2EUNION\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V0s)) \vee (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V1t)))))) \quad (35)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in (2^{A\_27a}). (\forall V2x \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ (ap\ (ap\ (c\_2Epred\_set\_2EINTER\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V1t)))))) \quad (36)$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1u \in (2^{ty\_2Erealax\_2Ereal}). \\
& ((p (ap (ap (c\_2Etopology\_2Eclosed\_in ty\_2Erealax\_2Ereal) ( \\
& ap (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\
& c\_2Ereal\_topology\_2Eeuclidean) V1u)) V0s)) \Leftrightarrow (\exists V2t \in ( \\
& 2^{ty\_2Erealax\_2Ereal}).((p (ap c\_2Ereal\_topology\_2EClosed \\
& V2t)) \wedge (V0s = (ap (ap (c\_2Epred\_set\_2EINTER ty\_2Erealax\_2Ereal) \\
& V1u) V2t))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).((p (ap c\_2Ereal\_topology\_2Econnected \\
& V0s)) \Leftrightarrow (\neg(\exists V1e1 \in (2^{ty\_2Erealax\_2Ereal}).(\exists V2e2 \in \\
& (2^{ty\_2Erealax\_2Ereal}).((p (ap c\_2Ereal\_topology\_2EClosed \\
& V1e1)) \wedge ((p (ap c\_2Ereal\_topology\_2EClosed V2e2)) \wedge ((p (ap (ap \\
& (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) V0s) (ap (ap (c\_2Epred\_set\_2EUNION \\
& ty\_2Erealax\_2Ereal) V1e1) V2e2))) \wedge ((ap (ap (c\_2Epred\_set\_2EINTER \\
& ty\_2Erealax\_2Ereal) (ap (ap (c\_2Epred\_set\_2EINTER ty\_2Erealax\_2Ereal) \\
& V1e1) V2e2)) V0s) = (c\_2Epred\_set\_2EEMPTY ty\_2Erealax\_2Ereal)) \wedge \\
& ((\neg((ap (ap (c\_2Epred\_set\_2EINTER ty\_2Erealax\_2Ereal) V1e1) \\
& V0s) = (c\_2Epred\_set\_2EEMPTY ty\_2Erealax\_2Ereal))) \wedge (\neg((ap \\
& (ap (c\_2Epred\_set\_2EINTER ty\_2Erealax\_2Ereal) V2e2) V0s) = ( \\
& c\_2Epred\_set\_2EEMPTY ty\_2Erealax\_2Ereal))))))))))
\end{aligned} \tag{38}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{39}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow (\neg(p V0A)) \Rightarrow False)) \tag{40}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \tag{41}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \tag{42}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False)) \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg( \\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge ((p \ V0p) \vee (\neg(p \ V2r)))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge ((p \ V0p) \vee (\neg(p \ V2r))) \wedge ( \\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{48}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{49}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{50}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}). ((p \ (ap \ c\_2Ereal\_topology\_2Econnected \\
& \ V0s)) \Leftrightarrow (\neg(\exists V1e1 \in (2^{ty\_2Erealax\_2Ereal}). (\exists V2e2 \in \\
& (2^{ty\_2Erealax\_2Ereal}). ((p \ (ap \ (ap \ (c\_2Etopology\_2Eclosed\_in \\
& \ ty\_2Erealax\_2Ereal) \ (ap \ (ap \ (c\_2Ereal\_topology\_2Esubtopology \\
& \ ty\_2Erealax\_2Ereal) \ c\_2Ereal\_topology\_2Eeuclidean) \ V0s)) \\
& \ V1e1)) \wedge ((p \ (ap \ (ap \ (c\_2Etopology\_2Eclosed\_in \ ty\_2Erealax\_2Ereal) \\
& \ (ap \ (ap \ (c\_2Ereal\_topology\_2Esubtopology \ ty\_2Erealax\_2Ereal) \\
& \ c\_2Ereal\_topology\_2Eeuclidean) \ V0s)) \ V2e2)) \wedge ((p \ (ap \ (ap \ (c\_2Epred\_set\_2ESUBSET \\
& \ ty\_2Erealax\_2Ereal) \ V0s) \ (ap \ (ap \ (c\_2Epred\_set\_2EUNION \ ty\_2Erealax\_2Ereal) \\
& \ V1e1) \ V2e2))) \wedge (((ap \ (ap \ (c\_2Epred\_set\_2EINTER \ ty\_2Erealax\_2Ereal) \\
& \ V1e1) \ V2e2) = (c\_2Epred\_set\_2EEMPTY \ ty\_2Erealax\_2Ereal)) \wedge ( \\
& \ (\neg(V1e1 = (c\_2Epred\_set\_2EEMPTY \ ty\_2Erealax\_2Ereal))) \wedge (\neg( \\
& \ V2e2 = (c\_2Epred\_set\_2EEMPTY \ ty\_2Erealax\_2Ereal))))))))))
\end{aligned}$$