

thm_2Ereal__topology_2ECONNECTED__IMP__PERFECT
(TMTbBVnTRZap-
WQTJF25fd4HPtcDEUEXpxNF)

October 26, 2020

Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_2IN$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0x \in A.27a.(\lambda V1f \in (2^{A-27a}).(ap V1f V0x)))$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V1t \in 2.V1t)) (\lambda V2t \in 2.V2t))$

Definition 7 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epair_2EABS_prod A.27a A.27b \in ((ty_2Epair_2Eprod A.27a A.27b)^{(2^{A-27b})^{A-27a}}) \tag{2}$$

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c_2Epair_2EABS_prod A.27a A.27b) (V0x V1y))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (3)$$

Definition 10 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap\ (c_2Emin_2E40\ t))$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (4)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (5)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (6)$$

Definition 11 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ t))$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (7)$$

Definition 12 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 13 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E21\ 2))\ (\lambda V0t \in 2.V0t)$.

Definition 14 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t))\ c_2Ebool_2E21)$

Definition 15 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 16 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Emin_2E40\ t))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (8)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (10)$$

Definition 17 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (11)$$

Definition 18 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2E$

Definition 19 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Etopology_2Etopology A0) \quad (12)$$

Let $c_2Etopology_2Eopen_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Etopology_2Eopen_in A_27a \in ((2^{(2^{A_27a})})^{(ty_2Etopology_2Etopology A_27a)}) \quad (13)$$

Definition 20 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40$

Definition 21 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A_27a})}). (ap (c_2Epred_s$

Definition 22 We define $c_2Etopology_2Etopspace$ to be $\lambda A_27a : \iota. \lambda V0top \in (ty_2Etopology_2Etopology$

Definition 23 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2E$

Definition 24 We define $c_2Etopology_2Eclosed_in$ to be $\lambda A_27a : \iota. \lambda V0top \in (ty_2Etopology_2Etopology$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)}) \quad (14)$$

Definition 25 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}). (ap (c_2Ebool_2E$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Etopology_2Etopology A_27a \in ((ty_2Etopology_2Etopology A_27a)^{(2^{(2^{A_27a})})}) \quad (15)$$

Definition 26 We define $c_2Ereal_topology_2Eeuclidean$ to be $(ap (c_2Etopology_2Etopology ty_2Erealax$

Definition 27 We define $c_2Ereal_topology_2Esubtopology$ to be $\lambda A_27a : \iota. \lambda V0top \in (ty_2Etopology_2Etopology$

Definition 28 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c$

Definition 29 We define $c_2Ereal_topology_2Econnected$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}). (ap c_2Ebool_2E$

Definition 30 We define $c_2Ereal_topology_2Elimit_point_of$ to be $\lambda V0x \in ty_2Erealax_2Ereal. \lambda V1s \in ($

Let $c_2Ereal_topology_2Ecball : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2Ecball \in ((2^{ty_2Erealax_2Ereal})(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)) \quad (16)$$

Definition 31 We define $c_2Epred_set_2EUNIV$ to be $\lambda A.27a : \iota.(\lambda V0x \in A.27a.c_2Ebool_2ET)$.

Definition 32 We define $c_2Ereal_topology_2EClosed$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ c_2Ereal_topo$

Assume the following.

$$True \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (27)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (28)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\forall V1x \in A.27a.(p(ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p(ap V0P V2x)))))) \quad (31)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\exists V1x \in A.27a.(p(ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p(ap V0P V2x)))))) \quad (32)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\ & 2^{A.27a}).(((p V0P) \wedge (\forall V2x \in A.27a.(p(ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in \\ & A.27a.((p V0P) \wedge (p(ap V1Q V3x)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\ & 2^{A.27a}).(((p V0P) \vee (\exists V2x \in A.27a.(p(ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in \\ & A.27a.((p V0P) \vee (p(ap V1Q V3x)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ & 2.((\exists V2x \in A.27a.((p(ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in \\ & A.27a.(p(ap V0P V3x)) \wedge (p V1Q)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a. (p (ap V1Q V3x))))))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. ((\neg((p V0A) \Rightarrow (p V1B))) \Leftrightarrow ((p V0A) \wedge (\neg(p V1B))))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee (\neg(p V1B)))))) \wedge (((\neg(p V0A) \vee (p V1B)) \Leftrightarrow ((\neg(p V0A) \wedge (\neg(p V1B))))))) \quad (40)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (41)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x.27 \in 2. (\forall V2y \in 2. (\forall V3y.27 \in 2. (((((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \quad (42)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0a \in A.27a. (\exists V1x \in A.27a. (V1x = V0a))) \quad (43)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\forall V0P \in ((2^{A.27b})^{A.27a}). ((\forall V1x \in A.27a. (\exists V2y \in A.27b. (p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A.27b^{A.27a}). (\forall V4x \in A.27a. (p (ap (ap V0P V4x) (ap V3f V4x))))))) \quad (44)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN \\ & A_27a) V2x) V0s)) \Leftrightarrow (p (ap (ap (c_2Ebool_2EIN\ A_27a) V2x) V1t))))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg (p (ap (ap \\ & (c_2Ebool_2EIN\ A_27a) V0x) (c_2Epred_set_2EEMPTY\ A_27a)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). (\forall V2x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN\ A_27a) \\ & V2x) (ap (ap (c_2Epred_set_2EINTER\ A_27a) V0s) V1t))) \Leftrightarrow ((p (ap \\ & (ap (c_2Ebool_2EIN\ A_27a) V2x) V0s)) \wedge (p (ap (ap (c_2Ebool_2EIN \\ & A_27a) V2x) V1t))))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ & A_27a. (\forall V2s \in (2^{A_27a}). ((p (ap (ap (c_2Ebool_2EIN\ A_27a) \\ & V0x) (ap (ap (c_2Epred_set_2EINSERT\ A_27a) V1y) V2s))) \Leftrightarrow ((V0x = \\ & V1y) \vee (p (ap (ap (c_2Ebool_2EIN\ A_27a) V0x) V2s))))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ & A_27a. ((p (ap (ap (c_2Ebool_2EIN\ A_27a) V0x) (ap (ap (c_2Epred_set_2EINSERT \\ & A_27a) V1y) (c_2Epred_set_2EEMPTY\ A_27a)))) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\ & ((p (ap (ap (c_2Erealax_2Ereal_lte\ V0x) V1y)) \Rightarrow (p (ap (ap (c_2Ereal_2Ereal_lte \\ & V0x) V1y)))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1e \in ty_2Erealax_2Ereal. \\ & ((p (ap (ap (c_2Ebool_2EIN\ ty_2Erealax_2Ereal) V0x) (ap (c_2Ereal_topology_2Ecball \\ & (ap (ap (c_2Epair_2E2C\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal) \\ & V0x) V1e)))) \Leftrightarrow (p (ap (ap (c_2Ereal_2Ereal_lte\ (ap (c_2Ereal_2Ereal_of_num \\ & c_2Enum_2E0) V1e)))))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).((p (ap c_2Ereal_topology_2EOpen \\
& V0s)) \Leftrightarrow (\forall V1x \in ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN \\
& ty_2Erealax_2Ereal) V1x) V0s)) \Rightarrow (\exists V2e \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V2e)) \wedge (p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) \\
& (ap c_2Ereal_topology_2Ecball (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal) V1x) V2e)))) V0s)))))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1u \in (2^{ty_2Erealax_2Ereal}). \\
& ((p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) (ap \\
& (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& c_2Ereal_topology_2Eeuclidean) V1u)) V0s)) \Leftrightarrow (\exists V2t \in (\\
& 2^{ty_2Erealax_2Ereal}).((p (ap c_2Ereal_topology_2EOpen V2t)) \wedge \\
& (V0s = (ap (ap (c_2Epred_set_2EINTER ty_2Erealax_2Ereal) V1u) \\
& V2t)))))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1u \in (2^{ty_2Erealax_2Ereal}). \\
& ((p (ap (ap (c_2Etopology_2Eclosed_in ty_2Erealax_2Ereal) (\\
& ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& c_2Ereal_topology_2Eeuclidean) V1u)) V0s)) \Leftrightarrow (\exists V2t \in (\\
& 2^{ty_2Erealax_2Ereal}).((p (ap c_2Ereal_topology_2Eclosed \\
& V2t)) \wedge (V0s = (ap (ap (c_2Epred_set_2EINTER ty_2Erealax_2Ereal) \\
& V1u) V2t)))))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).((p (ap c_2Ereal_topology_2Econnected \\
& V0s)) \Leftrightarrow (\forall V1t \in (2^{ty_2Erealax_2Ereal}).(((p (ap (ap (c_2Etopology_2Eopen_in \\
& ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& ty_2Erealax_2Ereal) c_2Ereal_topology_2Eeuclidean) V0s)) \\
& V1t)) \wedge (p (ap (ap (c_2Etopology_2Eclosed_in ty_2Erealax_2Ereal) \\
& (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& c_2Ereal_topology_2Eeuclidean) V0s)) V1t))) \Rightarrow ((V1t = (c_2Epred_set_2EEMPTY \\
& ty_2Erealax_2Ereal)) \vee (V1t = V0s)))))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1e \in ty_2Erealax_2Ereal. \\
& (p (ap c_2Ereal_topology_2Eclosed (ap c_2Ereal_topology_2Ecball \\
& (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V0x) V1e)))))))))
\end{aligned} \tag{56}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (57)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (58)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (59)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (60)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (62)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (63)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (64)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (65)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (66)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (67)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (68)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (69)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (70)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (71)$$

Theorem 1

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}). (\forall V1x \in ty_2Erealax_2Ereal. ((p (ap c_2Ereal_topology_2Econnected V0s)) \wedge ((\neg(\exists V2a \in ty_2Erealax_2Ereal. (V0s = (ap (ap (c_2Epred_set_2EINSERT ty_2Erealax_2Ereal) V2a) (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal)))) \wedge (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V1x) V0s)))) \Rightarrow (p (ap (ap c_2Ereal_topology_2Elimit_point_of V1x) V0s))))))$$