

# thm\_2Ereal\_\_topology\_2ECONNECTED\_\_MONOTONE\_\_QUOTIE (TMcM8ZVNLoMgw6jya8x2mWH1ZeTZyw577j3)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ if  $(\exists x \in A.p (ap P x))$  **then** (the  $(\lambda x.x \in A \wedge p$   
of type  $\iota \Rightarrow \iota$ ).

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$   
of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda A.\lambda 27c : \iota.(\lambda V0f \in ((A.\lambda 27c^{A.\lambda 27b})^{A.\lambda 27a})$

**Definition 4** We define  $c\_2Ecombin\_2EC$  to be  $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda A.\lambda 27c : \iota.(\lambda V0f \in ((A.\lambda 27c^{A.\lambda 27b})^{A.\lambda 27a})$

**Definition 5** We define  $c\_2Ebool\_2E\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 6** We define  $c\_2Ebool\_2E\_2E21$  to be  $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A.\lambda 27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A.\lambda 27a})$

**Definition 7** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda A.\lambda 27c : \iota.\lambda V0f \in (A.\lambda 27b^{A.\lambda 27c}).\lambda V1g$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (1)$$

**Definition 8** We define  $c\_2Ebool\_2E\_2E3F$  to be  $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A.\lambda 27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Etopology\_2Etopology A0) \quad (2)$$

Let  $c\_2Etopology\_2Eopen\_in : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.\lambda 27a.nonempty A.\lambda 27a \Rightarrow c\_2Etopology\_2Eopen\_in A.\lambda 27a \in ((2^{(2^{A.\lambda 27a})})_{(ty\_2Etopology\_2Etopology A.\lambda 27a)}) \quad (3)$$

**Definition 9** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 10** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V0t))$   
Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (4)$$

**Definition 11** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epair\_2EABS\_prod A\_27a A\_27b) (V0x V1y))$   
Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \end{aligned} \quad (5)$$

**Definition 12** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 13** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap (c\_2Epred\_set\_2EGSPEC A\_27a) V0P)$

**Definition 14** We define  $c\_2Etopology\_2Etopspace$  to be  $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology A\_27a)$

**Definition 15** We define  $c\_2Ebool\_2E2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 16** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F))$

**Definition 17** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_2F) (V0s V1t))$

**Definition 18** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_2F) (V0s V1t))$

**Definition 19** We define  $c\_2Etopology\_2Eclosed\_in$  to be  $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology A\_27a)$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty ty\_2Erealax\_2Ereal \quad (6)$$

**Definition 20** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E2F)$ .

**Definition 21** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_2F) (V0s V1t))$

**Definition 22** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V0t))$

**Definition 23** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_2F) (V0s V1t))$

Let  $c\_2Ereal\_topology\_2EDist : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \quad (7)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (8)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax}) \quad (9)$$

**Definition 24** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E40\ (t$

Let  $c\_2Erealax\_2Etreallt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreallt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal)}) \quad (10)$$

**Definition 25** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in omega \quad (11)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (12)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (13)$$

**Definition 26** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (14)$$

**Definition 27** We define  $c\_2Ereal\_topology\_2EOpen$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap\ (c\_2Ebool\_2E2$

**Definition 28** We define  $c\_2Ereal\_topology\_2Econnected$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap\ c\_2Ebool\_2E2$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^A - 27a)}})) \quad (15)$$

**Definition 29** We define  $c\_2Ereal\_topology\_2Eeuclidean$  to be  $(ap\ (c\_2Etopology\_2Etopology\ ty\_2Erealax$

**Definition 30** We define  $c\_2Ereal\_topology\_2Esubtopology$  to be  $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology)$

**Definition 31** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in$

**Definition 32** We define  $c\_2Ereal\_topology\_2Econtinuous\_on$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Ereal})$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \vee \neg(p V0t)))) \quad (19)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow \neg(p V0t))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))) \quad (24)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (25)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \quad (26)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (27)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (29)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p\ V0P) \wedge (\forall V2x \in A\_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A\_27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x))))))) \quad (30)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A\_27a}).(((\forall V2x \in A\_27a.((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in A\_27a.(p\ (ap\ V1P\ V3x))) \vee (p\ V0Q)))))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ((p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C)))))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V1B) \wedge ((p\ V2C) \vee (p\ V0A))) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A)))))) \quad (34)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (35)$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \Rightarrow (36)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1a \in A_{.27a}.((\exists V2x \in A_{.27a}.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \Rightarrow (37)$$

Assume the following.

$$(\forall V0r \in 2.(\forall V1p \in 2.(\forall V2q \in 2.(((p V1p) \wedge (p V2q)) \Rightarrow (p V0r)) \Leftrightarrow ((p V1p) \Rightarrow ((p V2q) \Rightarrow (p V0r)))))) \Rightarrow (38)$$

Assume the following.

$$(\forall V0r \in 2.(\forall V1p \in 2.(\forall V2q \in 2.(((p V1p) \wedge (p V2q)) \Rightarrow (p V0r)) \Leftrightarrow ((p V2q) \Rightarrow ((p V1p) \Rightarrow (p V0r)))))) \Rightarrow (39)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1P \in (2^{A_{.27a}}).(p (ap (ap (c_{.2}Epred_{.set}_{.2}ESUBSET A_{.27a}) (ap (c_{.2}Epred_{.set}_{.2}EGSPEC A_{.27a} A_{.27a}) (\lambda V2x \in A_{.27a}.(ap (ap (c_{.2}Epair_{.2E}_{.2C} A_{.27a} 2) V2x) (ap (ap c_{.2}Ebool_{.2E}_{.2F}_{.5C} (ap (ap (c_{.2}Ebool_{.2EIN} A_{.27a}) V2x) V0s)) (ap V1P V2x)))))) V0s)))) \Rightarrow (40)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.(\forall V2a \in A_{.27a}.(\forall V3b \in A_{.27b}.(((ap (ap (c_{.2}Epair_{.2E}_{.2C} A_{.27a} A_{.27b}) V0x) V1y) = (ap (ap (c_{.2}Epair_{.2E}_{.2C} A_{.27a} A_{.27b}) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \Rightarrow (41)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in (2^{A_{.27a}}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_{.27a}.((p (ap (ap (c_{.2}Ebool_{.2EIN} A_{.27a}) V2x) V0s)) \Leftrightarrow (p (ap (ap (c_{.2}Ebool_{.2EIN} A_{.27a}) V2x) V1t)))))) \Rightarrow (42)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0f \in ((ty_{.2}Epair_{.2E}prod A_{.27a} 2)^{A_{.27b}}).(\forall V1v \in A_{.27a}.((p (ap (ap (c_{.2}Ebool_{.2EIN} A_{.27a}) V1v) (ap (c_{.2}Epred_{.set}_{.2}EGSPEC A_{.27a} A_{.27b}) V0f)) \Leftrightarrow (\exists V2x \in A_{.27b}.((ap (ap (c_{.2}Epair_{.2E}_{.2C} A_{.27a} 2) V1v) c_{.2}Ebool_{.2ET}) = (ap V0f V2x)))))) \Rightarrow (43)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (p (ap (ap (c\_2Epred\_set\_2ESUBSET\ A\_27a)\ V0s)\ V0s))) \quad (44)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0y \in A\_27b. (\forall V1s \in (2^{A\_27a}). (\forall V2f \in (A\_27b^{A\_27a}). \\ & ((p (ap (ap (c\_2Ebool\_2EIN\ A\_27b)\ V0y)\ (ap (ap (c\_2Epred\_set\_2EIMAGE \\ & \quad A\_27a\ A\_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A\_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\ & \quad (p (ap (ap (c\_2Ebool\_2EIN\ A\_27a)\ V3x)\ V1s))))))) \end{aligned} \quad (45)$$

Assume the following.

$$(\forall V0P \in 2. (\forall V1Q \in 2. ((p\ V0P) \Leftrightarrow (p\ V1Q)) \Rightarrow ((p\ V0P) \Rightarrow (p\ V1Q)))) \quad (46)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0top \in (ty\_2Etopology\_2Etopology \\ & \quad A\_27a). (\forall V1s \in (2^{A\_27a}). (\forall V2t \in (2^{A\_27a}). ((p ( \\ & ap (ap (c\_2Etopology\_2Eopen\_in\ A\_27a)\ (ap (ap (c\_2Ereal\_topology\_2Esubtopology \\ & \quad A\_27a)\ V0top)\ V1s))\ V2t)) \Rightarrow (p (ap (ap (c\_2Epred\_set\_2ESUBSET\ A\_27a) \\ & \quad V2t)\ V1s)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0top \in (ty\_2Etopology\_2Etopology \\ & \quad A\_27a). (\forall V1s \in (2^{A\_27a}). (\forall V2t \in (2^{A\_27a}). ((p ( \\ & ap (ap (c\_2Etopology\_2Eclosed\_in\ A\_27a)\ (ap (ap (c\_2Ereal\_topology\_2Esubtopology \\ & \quad A\_27a)\ V0top)\ V1s))\ V2t)) \Rightarrow (p (ap (ap (c\_2Epred\_set\_2ESUBSET\ A\_27a) \\ & \quad V2t)\ V1s)))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). (\forall V1s \in \\ & \quad (2^{ty\_2Erealax\_2Ereal}). (\forall V2t \in (2^{ty\_2Erealax\_2Ereal}). \\ & ((p (ap (ap\ c\_2Ereal\_topology\_2Econtinuous\_on\ V0f)\ V1s)) \wedge \\ & (p (ap (ap (c\_2Epred\_set\_2ESUBSET\ ty\_2Erealax\_2Ereal)\ V2t)\ V1s))) \Rightarrow \\ & (p (ap (ap\ c\_2Ereal\_topology\_2Econtinuous\_on\ V0f)\ V2t)))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1s \in \\
& \quad (2^{ty\_2Erealax\_2Ereal}).(\forall V2t \in (2^{ty\_2Erealax\_2Ereal}). \\
& \quad ((p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) (ap \\
& \quad (ap (c\_2Epred\_set\_2EIMAGE ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& \quad V0f) V1s)) V2t)) \wedge (\forall V3u \in (2^{ty\_2Erealax\_2Ereal}).((p (ap \\
& \quad (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) V3u) V2t)) \Rightarrow \\
& \quad ((p (ap (ap (c\_2Etopology\_2Eopen\_in ty\_2Erealax\_2Ereal) (ap \\
& \quad (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\
& \quad c\_2Ereal\_topology\_2Eeuclidean) V1s)) (ap (c\_2Epred\_set\_2EGSPEC \\
& \quad ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) (\lambda V4x \in ty\_2Erealax\_2Ereal. \\
& \quad (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal 2) V4x) (ap (ap c\_2Ebool\_2E\_2F\_5C \\
& \quad (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V4x) V1s)) (ap (ap ( \\
& \quad c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) (ap V0f V4x)) V3u)))))) \Leftrightarrow ( \\
& \quad p (ap (ap (c\_2Etopology\_2Eopen\_in ty\_2Erealax\_2Ereal) (ap (ap \\
& \quad (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Eeuclidean) \\
& \quad V2t)) V3u)))))) \Rightarrow (p (ap (ap c\_2Ereal\_topology\_2Econtinuous\_on \\
& \quad V0f) V1s))))))
\end{aligned} \tag{50}$$



Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1s \in \\
& \quad (2^{ty\_2Erealax\_2Ereal}).(\forall V2t \in (2^{ty\_2Erealax\_2Ereal}). \\
& \quad (\forall V3c \in (2^{ty\_2Erealax\_2Ereal}).(((p (ap (ap (c\_2Epred\_set\_2ESUBSET \\
& \quad ty\_2Erealax\_2Ereal) (ap (ap (c\_2Epred\_set\_2EIMAGE ty\_2Erealax\_2Ereal \\
& \quad ty\_2Erealax\_2Ereal) V0f) V1s)) V2t)) \wedge ((\forall V4u \in (2^{ty\_2Erealax\_2Ereal}). \\
& \quad ((p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) V4u) \\
& \quad V2t)) \Rightarrow ((p (ap (ap (c\_2Etopology\_2Eopen\_in ty\_2Erealax\_2Ereal) \\
& \quad (ap (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\
& \quad c\_2Ereal\_topology\_2Eeuclidean) V1s)) (ap (c\_2Epred\_set\_2EGSPEC \\
& \quad ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) (\lambda V5x \in ty\_2Erealax\_2Ereal. \\
& \quad (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal 2) V5x) (ap (ap c\_2Ebool\_2E\_2F\_5C \\
& \quad (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V5x) V1s)) (ap (ap ( \\
& \quad c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) (ap V0f V5x)) V4u)))))) \Leftrightarrow ( \\
& \quad p (ap (ap (c\_2Etopology\_2Eopen\_in ty\_2Erealax\_2Ereal) (ap (ap \\
& \quad (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Eeuclidean) \\
& \quad V2t)) V4u)))) \wedge ((p (ap (ap (c\_2Etopology\_2Eopen\_in ty\_2Erealax\_2Ereal) \\
& \quad (ap (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\
& \quad c\_2Ereal\_topology\_2Eeuclidean) V2t)) V3c)) \vee (p (ap (ap (c\_2Etopology\_2Eclosed\_in \\
& \quad ty\_2Erealax\_2Ereal) (ap (ap (c\_2Ereal\_topology\_2Esubtopology \\
& \quad ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Eeuclidean) V2t)) \\
& \quad V3c)))) \Rightarrow (\forall V6u \in (2^{ty\_2Erealax\_2Ereal}).((p (ap (ap (c\_2Epred\_set\_2ESUBSET \\
& \quad ty\_2Erealax\_2Ereal) V6u) V3c)) \Rightarrow ((p (ap (ap (c\_2Etopology\_2Eopen\_in \\
& \quad ty\_2Erealax\_2Ereal) (ap (ap (c\_2Ereal\_topology\_2Esubtopology \\
& \quad ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Eeuclidean) (ap (c\_2Epred\_set\_2EGSPEC \\
& \quad ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) (\lambda V7x \in ty\_2Erealax\_2Ereal. \\
& \quad (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal 2) V7x) (ap (ap c\_2Ebool\_2E\_2F\_5C \\
& \quad (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V7x) V1s)) (ap (ap ( \\
& \quad c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) (ap V0f V7x)) V3c)))))) (ap \\
& \quad (c\_2Epred\_set\_2EGSPEC ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& \quad (\lambda V8x \in ty\_2Erealax\_2Ereal.(ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& \quad 2) V8x) (ap (ap c\_2Ebool\_2E\_2F\_5C (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\
& \quad V8x) (ap (c\_2Epred\_set\_2EGSPEC ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& \quad (\lambda V9x \in ty\_2Erealax\_2Ereal.(ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& \quad 2) V9x) (ap (ap c\_2Ebool\_2E\_2F\_5C (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\
& \quad V9x) V1s)) (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) (ap V0f V9x)) \\
& \quad V3c)))))) (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) (ap V0f \\
& \quad V8x)) V6u)))))) \Leftrightarrow (p (ap (ap (c\_2Etopology\_2Eopen\_in ty\_2Erealax\_2Ereal) \\
& \quad (ap (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\
& \quad c\_2Ereal\_topology\_2Eeuclidean) V3c)) V6u)))))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1s \in \\
& \quad (2^{ty\_2Erealax\_2Ereal}).(\forall V2t \in (2^{ty\_2Erealax\_2Ereal}). \\
& \quad ((p (ap (ap c\_2Ereal\_topology\_2Econtinuous\_on V0f) V1s)) \wedge \\
& \quad ((ap (ap (c\_2Epred\_set\_2EIMAGE ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& \quad \quad V0f) V1s) = V2t) \wedge ((\forall V3u \in (2^{ty\_2Erealax\_2Ereal}).((p (ap \\
& \quad (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) V3u) V2t)) \Rightarrow \\
& \quad ((p (ap (ap (c\_2Etopology\_2Eopen\_in ty\_2Erealax\_2Ereal) (ap \\
& \quad (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\
& \quad c\_2Ereal\_topology\_2Euclidean) V1s)) (ap (c\_2Epred\_set\_2EGSPEC \\
& \quad ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) (\lambda V4x \in ty\_2Erealax\_2Ereal. \\
& \quad (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal 2) V4x) (ap (ap c\_2Ebool\_2E\_2F\_5C \\
& \quad (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V4x) V1s)) (ap (ap ( \\
& \quad c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) (ap V0f V4x)) V3u)))))) \Leftrightarrow ( \\
& \quad p (ap (ap (c\_2Etopology\_2Eopen\_in ty\_2Erealax\_2Ereal) (ap (ap \\
& \quad (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Euclidean) \\
& \quad V2t)) V3u)))) \wedge ((\forall V5y \in ty\_2Erealax\_2Ereal.((p (ap (ap \\
& \quad (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V5y) V2t)) \Rightarrow (p (ap c\_2Ereal\_topology\_2Econnected \\
& \quad (ap (c\_2Epred\_set\_2EGSPEC ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& \quad (\lambda V6x \in ty\_2Erealax\_2Ereal.(ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& \quad 2) V6x) (ap (ap c\_2Ebool\_2E\_2F\_5C (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\
& \quad V6x) V1s)) (ap (ap (c\_2Emin\_2E\_3D ty\_2Erealax\_2Ereal) (ap V0f V6x)) \\
& \quad V5y)))))) \wedge (p (ap c\_2Ereal\_topology\_2Econnected V2t)))))) \Rightarrow \\
& \quad (p (ap c\_2Ereal\_topology\_2Econnected V1s))))))
\end{aligned} \tag{52}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{53}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{56}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \Leftrightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee ((\neg( \\
& p \ V2r)) \vee (\neg(p \ V1q)))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V0p)))) \wedge ((p \ V2r) \vee \\
& ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge (( \\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{62}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{63}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{64}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V0p)))) \tag{65}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{66}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \tag{67}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1s \in \\
& \quad (2^{ty\_2Erealax\_2Ereal}).(\forall V2t \in (2^{ty\_2Erealax\_2Ereal}). \\
& \quad (\forall V3c \in (2^{ty\_2Erealax\_2Ereal}).((((ap (ap (c\_2Epred\_set\_2EIMAGE \\
& \quad ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) V0f) V1s) = V2t) \wedge ((\forall V4u \in \\
& \quad (2^{ty\_2Erealax\_2Ereal}).((p (ap (ap (c\_2Epred\_set\_2ESUBSET \\
& \quad ty\_2Erealax\_2Ereal) V4u) V2t)) \Rightarrow ((p (ap (ap (c\_2Etopology\_2Eopen\_in \\
& \quad ty\_2Erealax\_2Ereal) (ap (ap (c\_2Ereal\_topology\_2Esubtopology \\
& \quad ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Eeuclidean) V1s)) \\
& \quad (ap (c\_2Epred\_set\_2EGSPEC ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& \quad (\lambda V5x \in ty\_2Erealax\_2Ereal.(ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& \quad 2) V5x) (ap (ap c\_2Ebool\_2E\_2F\_5C (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\
& \quad V5x) V1s)) (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) (ap V0f V5x)) \\
& \quad V4u)))))) \Leftrightarrow (p (ap (ap (c\_2Etopology\_2Eopen\_in ty\_2Erealax\_2Ereal) \\
& \quad (ap (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\
& \quad c\_2Ereal\_topology\_2Eeuclidean) V2t)) V4u)))) \wedge ((\forall V6y \in \\
& \quad ty\_2Erealax\_2Ereal.((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\
& \quad V6y) V2t)) \Rightarrow (p (ap c\_2Ereal\_topology\_2Econnected (ap (c\_2Epred\_set\_2EGSPEC \\
& \quad ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) (\lambda V7x \in ty\_2Erealax\_2Ereal. \\
& \quad (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal 2) V7x) (ap (ap c\_2Ebool\_2E\_2F\_5C \\
& \quad (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V7x) V1s)) (ap (ap ( \\
& \quad c\_2Emin\_2E\_3D ty\_2Erealax\_2Ereal) (ap V0f V7x)) V6y)))))) \wedge \\
& \quad (((p (ap (ap (c\_2Etopology\_2Eopen\_in ty\_2Erealax\_2Ereal) (ap \\
& \quad (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\
& \quad c\_2Ereal\_topology\_2Eeuclidean) V2t)) V3c)) \vee (p (ap (ap (c\_2Etopology\_2Eclosed\_in \\
& \quad ty\_2Erealax\_2Ereal) (ap (ap (c\_2Ereal\_topology\_2Esubtopology \\
& \quad ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Eeuclidean) V2t)) \\
& \quad V3c))) \wedge (p (ap c\_2Ereal\_topology\_2Econnected V3c)))))) \Rightarrow (p ( \\
& \quad ap c\_2Ereal\_topology\_2Econnected (ap (c\_2Epred\_set\_2EGSPEC \\
& \quad ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) (\lambda V8x \in ty\_2Erealax\_2Ereal. \\
& \quad (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal 2) V8x) (ap (ap c\_2Ebool\_2E\_2F\_5C \\
& \quad (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V8x) V1s)) (ap (ap ( \\
& \quad c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) (ap V0f V8x)) V3c)))))))))
\end{aligned}$$