

thm_2Ereal__topology_2ECONNECTED__OPEN__IN__EQ (TMQgvYgJogdVgJWWf1rtSmcuh4XnoFv73we)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).)(ap V1f V0x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).)(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 5 We define $c_2Ebool_2E_2EF$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E_2EF)$.

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2E_21 2) (c_2Emin_2E_3D (2^{A_27a}) (2^{A_27b})))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \tag{3}$$

Definition 17 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 18 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}). (ap\ (c_2Ebool_2E_2$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (13)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in \quad (14)$$

$$((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^A_27a)})})$$

Definition 19 We define $c_2Ereal_topology_2Eeuclidean$ to be $(ap\ (c_2Etopology_2Etopology\ ty_2Erealax_2Ereal$

Let $c_2Etopology_2Eopen_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Eopen_in\ A_27a \in \quad (15)$$

$$((2^{(2^A_27a)})^{(ty_2Etopology_2Etopology\ A_27a)})$$

Definition 20 We define $c_2Ereal_topology_2Esubtopology$ to be $\lambda A_27a : \iota. \lambda V0top \in (ty_2Etopology_2Etopology$

Definition 21 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_2$

Definition 22 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ ($

Definition 23 We define $c_2Ereal_topology_2Econnected$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}). (ap\ c_2Ebool_2E_2$

Definition 24 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2$

Definition 25 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^A_27a)}). (ap\ (c_2Epred_s$

Definition 26 We define $c_2Etopology_2Etopspace$ to be $\lambda A_27a : \iota. \lambda V0top \in (ty_2Etopology_2Etopology$

Definition 27 We define $c_2Etopology_2Eclosed_in$ to be $\lambda A_27a : \iota. \lambda V0top \in (ty_2Etopology_2Etopology$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (21)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (22)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\forall V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(\\ & p V0A)) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ & (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in \\ & 2.(((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\ & (2^{A.27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A.27a.((p (ap (ap (c.2Ebool.2EIN \\ & A.27a) V2x) V0s)) \Leftrightarrow (p (ap (ap (c.2Ebool.2EIN A.27a) V2x) V1t)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg (p (ap (ap (c_2Ebool_2EIN\ A_27a)\ V0x) (c_2Epred_set_2EEMPTY\ A_27a)))))) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (p (ap (ap (c_2Epred_set_2ESUBSET\ A_27a)\ V0s)\ V0s))) \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ (2^{A_27a}). (\forall V2x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN\ A_27a) \\ V2x) (ap (ap (c_2Epred_set_2EUNION\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p (ap \\ (ap (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \vee (p (ap (ap (c_2Ebool_2EIN \\ A_27a)\ V2x)\ V1t))))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ (2^{A_27a}). (\forall V2x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN\ A_27a) \\ V2x) (ap (ap (c_2Epred_set_2EINTER\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p (ap \\ (ap (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \wedge (p (ap (ap (c_2Ebool_2EIN \\ A_27a)\ V2x)\ V1t))))))) \end{aligned} \quad (33)$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}). ((ap (c_2Etopology_2Etopspace\ ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology\ ty_2Erealax_2Ereal)\ c_2Ereal_topology_2Eeuclidean)\ V0s)) = V0s)) \quad (34)$$

Assume the following.

$$\begin{aligned} (\forall V0s \in (2^{ty_2Erealax_2Ereal}). ((p (ap\ c_2Ereal_topology_2Econnected \\ V0s)) \Leftrightarrow (\neg (\exists V1e1 \in (2^{ty_2Erealax_2Ereal}). (\exists V2e2 \in \\ (2^{ty_2Erealax_2Ereal}). ((p (ap (ap (c_2Etopology_2Eopen_in \\ ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\ ty_2Erealax_2Ereal)\ c_2Ereal_topology_2Eeuclidean)\ V0s)) \\ V1e1)) \wedge ((p (ap (ap (c_2Etopology_2Eopen_in\ ty_2Erealax_2Ereal) \\ (ap (ap (c_2Ereal_topology_2Esubtopology\ ty_2Erealax_2Ereal) \\ c_2Ereal_topology_2Eeuclidean)\ V0s))\ V2e2)) \wedge ((p (ap (ap (c_2Epred_set_2ESUBSET \\ ty_2Erealax_2Ereal)\ V0s) (ap (ap (c_2Epred_set_2EUNION\ ty_2Erealax_2Ereal) \\ V1e1)\ V2e2))) \wedge (((ap (ap (c_2Epred_set_2EINTER\ ty_2Erealax_2Ereal) \\ V1e1)\ V2e2) = (c_2Epred_set_2EEMPTY\ ty_2Erealax_2Ereal)) \wedge (\\ (\neg (V1e1 = (c_2Epred_set_2EEMPTY\ ty_2Erealax_2Ereal))) \wedge (\neg (\\ V2e2 = (c_2Epred_set_2EEMPTY\ ty_2Erealax_2Ereal))))))))))))) \end{aligned} \quad (35)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (40)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (41)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (42)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (43)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (47)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0top \in (ty_2Etopology_2Etopology \\ A_27a).(\forall V1s \in (2^{A_27a}).((p (ap (ap (c_2Etopology_2Eopen_in \\ A_27a) V0top) V1s)) \Leftrightarrow ((p (ap (ap (c_2Epred_set_2ESUBSET A_27a) \\ V1s) (ap (c_2Etopology_2Etopspace A_27a) V0top))) \wedge (p (ap (ap (\\ c_2Etopology_2Eclosed_in A_27a) V0top) (ap (ap (c_2Epred_set_2EDIFF \\ A_27a) (ap (c_2Etopology_2Etopspace A_27a) V0top)) V1s))))))))) \end{aligned} \quad (48)$$

Theorem 1

$$\begin{aligned} (\forall V0s \in (2^{ty_2Erealax_2Ereal}).((p (ap c_2Ereal_topology_2Econnected \\ V0s)) \Leftrightarrow (\neg(\exists V1e1 \in (2^{ty_2Erealax_2Ereal}).(\exists V2e2 \in \\ (2^{ty_2Erealax_2Ereal}).((p (ap (ap (c_2Etopology_2Eopen_in \\ ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\ ty_2Erealax_2Ereal) c_2Ereal_topology_2Eeuclidean) V0s)) \\ V1e1)) \wedge ((p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) \\ (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\ c_2Ereal_topology_2Eeuclidean) V0s)) V2e2)) \wedge (((ap (ap (c_2Epred_set_2EUNION \\ ty_2Erealax_2Ereal) V1e1) V2e2) = V0s) \wedge (((ap (ap (c_2Epred_set_2EINTER \\ ty_2Erealax_2Ereal) V1e1) V2e2) = (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal))) \wedge \\ ((\neg(V1e1 = (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal))) \wedge (\neg \\ (V2e2 = (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal)))))))))))))) \end{aligned}$$