

thm_2Ereal__topology_2ECONNECTED__SEGMENT (TMF244MT9DLSScs1zf6AcahWdAiBLfGkb2h)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ecombin_2ES$ to be $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda A.\lambda 27c : \iota.(\lambda V0f \in ((A.\lambda 27c^{A.\lambda 27b})^{A.\lambda 27a})$

Definition 4 We define $c_2Ecombin_2EC$ to be $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda A.\lambda 27c : \iota.(\lambda V0f \in ((A.\lambda 27c^{A.\lambda 27b})^{A.\lambda 27a})$

Definition 5 We define $c_2Ecombin_2EK$ to be $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.(\lambda V0x \in A.\lambda 27a.(\lambda V1y \in A.\lambda 27b.V0x))$

Definition 6 We define $c_2Ecombin_2EI$ to be $\lambda A.\lambda 27a : \iota.(ap (ap (c_2Ecombin_2ES A.\lambda 27a (A.\lambda 27a^{A.\lambda 27a}) A.\lambda 27a$

Definition 7 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 8 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A.\lambda 27a}).(ap (ap (c_2Emin_2E_3D (2^{A.\lambda 27a})$

Definition 9 We define $c_2Ecombin_2Eo$ to be $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda A.\lambda 27c : \iota.(\lambda V0f \in (A.\lambda 27b^{A.\lambda 27c}).\lambda V1g$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})\ ty_2Erealax_2Ereal) \tag{4}$$

Definition 10 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E40 (t$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal \\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \end{aligned} \quad (5)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (6)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal))} \quad (7)$$

Definition 11 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty$

Definition 12 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealm_inv : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealm_inv \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal \\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \end{aligned} \quad (8)$$

Definition 13 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal_ABS$

Let $c_2Erealax_2Etrealm_mul : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealm_mul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal \\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \end{aligned} \quad (9)$$

Definition 14 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Definition 15 We define c_2Ereal_2E2F to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.($

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND \\ A_27a A_27b \in (A_27b)^{(ty_2Epair_2Eprod A_27a A_27b)} \end{aligned} \quad (10)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST \\ A_27a A_27b \in (A_27a)^{(ty_2Epair_2Eprod A_27a A_27b)} \end{aligned} \quad (11)$$

Definition 16 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c)^{A_27a$

Definition 17 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Definition 18 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 19 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 20 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_21))$

Definition 21 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ V2t)\ V1t2)\ V0t1)))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (12)$$

Definition 22 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Epair_2EABS_prod\ V0x\ V1y)\ A_27a\ A_27b)$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (13)$$

Definition 23 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Ebool_2E_21\ 2)\ V0s\ V1t)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (14)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (15)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (16)$$

Definition 24 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 25 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (17)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (18)$$

Definition 26 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (19)$$

Definition 27 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 28 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Erealax_2Etreall_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (20)$$

Definition 29 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Let $c_2Ereal_topology_2Eopen_segment : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2Eopen_segment \in ((2^{ty_2Erealax_2Ereal})^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (21)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (22)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ENIL\ A.27a \in (ty_2Elist_2Elist\ A.27a) \quad (23)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ECONS\ A.27a \in (((ty_2Elist_2Elist\ A.27a)^{(ty_2Elist_2Elist\ A.27a)})^{A.27a}) \quad (24)$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (25)$$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (26)$$

Definition 30 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Definition 31 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Elist_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EHD\ A_27a \in (A_27a^{(ty_2Elist_2Elist\ A_27a)}) \quad (27)$$

Definition 32 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$.

Definition 33 We define $c_2Ereal_topology_2Eclosed_segment$ to be $\lambda V0l \in (ty_2Elist_2Elist\ (ty_2Epair_2Ereal))$.

Definition 34 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Ebool_2E21\ 2))$.

Definition 35 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2))\ (\lambda V2t \in 2)))$.

Definition 36 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Ebool_2E21\ 2))$.

Definition 37 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Ebool_2E21\ 2))$.

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (28)$$

Definition 38 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E40\ 2))))$.

Definition 39 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (c_2Ebool_2E21\ 2))$.

Definition 40 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E2F)$.

Definition 41 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap\ (c_2Ebool_2E21\ 2))$.

Definition 42 We define $c_2Ereal_topology_2Econnected$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ c_2Ebool_2E21\ 2)$.

Definition 43 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap\ (c_2Ebool_2E21\ 2))))))$.

Definition 44 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND))))$.

Assume the following.

$$True \quad (29)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee \neg(p\ V0t))) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (33)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \wedge ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (37)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (38)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (40)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (41)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (42)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\exists V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (43)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in (2^{A.27a}).((\forall V2x \in A.27a.((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A.27a.(p (ap V0P V3x))) \wedge (\forall V4x \in A.27a.(p (ap V1Q V4x))))))) \quad (44)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A.27a}).((\forall V2x \in A.27a.((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A.27a.(p (ap V1P V3x))) \vee (p V0Q)))))) \quad (45)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).((\forall V2x \in A.27a.((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a.(p (ap V1Q V3x))))))) \quad (46)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.((\neg((p V0A) \Rightarrow (p V1B))) \Leftrightarrow ((p V0A) \wedge (\neg(p V1B)))))) \quad (47)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))) \quad (48)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee ((p V1B) \wedge (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C)))))) \quad (49)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V1B) \wedge ((p V2C) \vee (p V0A))) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (50)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (51)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in 2.(((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \quad (52)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ & 2.(((\exists V2x \in A.27a.(p (ap V0P V2x))) \Rightarrow (p V1Q)) \Leftrightarrow (\forall V3x \in \\ & A.27a.((p (ap V0P V3x)) \Rightarrow (p V1Q)))))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\ & 2^{A.27a}).(((p V0P) \Rightarrow (\forall V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in \\ & A.27a.((p V0P) \Rightarrow (p (ap V1Q V3x)))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0r \in (2^{A.27a}).(\forall V1p \in \\ & (2^{A.27a}).(\forall V2q \in (2^{A.27a}).(((ap (ap (c.2Epred_set_2EINTER \\ & A.27a) V1p) V2q) = (ap (ap (c.2Epred_set_2EINTER A.27a) V2q) V1p)) \wedge \\ & (((ap (ap (c.2Epred_set_2EINTER A.27a) (ap (ap (c.2Epred_set_2EINTER \\ & A.27a) V1p) V2q)) V0r) = (ap (ap (c.2Epred_set_2EINTER A.27a) (\\ & ap (ap (c.2Epred_set_2EINTER A.27a) V1p) V2q)) V0r)) \wedge (((ap (ap \\ & (c.2Epred_set_2EINTER A.27a) (ap (ap (c.2Epred_set_2EINTER \\ & A.27a) V1p) V2q)) V0r) = (ap (ap (c.2Epred_set_2EINTER A.27a) (\\ & ap (ap (c.2Epred_set_2EINTER A.27a) V2q) V1p)) V0r)) \wedge (((ap (ap \\ & (c.2Epred_set_2EINTER A.27a) V1p) V1p) = V1p) \wedge ((ap (ap (c.2Epred_set_2EINTER \\ & A.27a) (ap (ap (c.2Epred_set_2EINTER A.27a) V1p) V1p)) V2q) = (\\ & ap (ap (c.2Epred_set_2EINTER A.27a) V1p) V2q)))))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((ap (c.2Ecombin_2EI \\ & A.27a) V0x) = V0x)) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\\ & \forall V0f \in (A.27b^{A.27a}).(((ap (ap (c.2Ecombin_2Eo A.27a A.27b \\ & A.27b) (c.2Ecombin_2EI A.27b)) V0f) = V0f) \wedge ((ap (ap (c.2Ecombin_2Eo \\ & A.27a A.27b A.27a) V0f) (c.2Ecombin_2EI A.27a)) = V0f))) \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\\ & \forall V0x \in A.27a.(\forall V1y \in A.27b.(\forall V2a \in A.27a.(\forall V3b \in \\ & A.27b.(((ap (ap (c.2Epair_2E_2C A.27a A.27b) V0x) V1y) = (ap (ap \\ & (c.2Epair_2E_2C A.27a A.27b) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\ & (2^{A.27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A.27a.((p (ap (ap (c.2Ebool_2EIN \\ & A.27a) V2x) V0s)) \Leftrightarrow (p (ap (ap (c.2Ebool_2EIN A.27a) V2x) V1t)))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ \forall V0f \in ((ty.2Epair.2Eprod\ A.27a\ 2)^{A.27b}).(\forall V1v \in \\ A.27a.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V1v)\ (ap\ (c.2Epred_set.2EGSPEC \\ A.27a\ A.27b)\ V0f)))) \Leftrightarrow (\exists V2x \in A.27b.((ap\ (ap\ (c.2Epair.2E.2C \\ A.27a\ 2)\ V1v)\ c.2Ebool.2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (60)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\neg(p\ (ap\ (ap \\ (c.2Ebool.2EIN\ A.27a)\ V0x)\ (c.2Epred_set.2EEMPTY\ A.27a)))))) \quad (61)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).((\exists V1x \in \\ A.27a.(p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V1x)\ V0s))) \Leftrightarrow (\neg(V0s = (c.2Epred_set.2EEMPTY \\ A.27a)))))) \end{aligned} \quad (62)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\ (2^{A.27a}).(\forall V2x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a) \\ V2x)\ (ap\ (ap\ (c.2Epred_set.2EUNION\ A.27a)\ V0s)\ V1t)))) \Leftrightarrow ((p\ (ap \\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V0s)) \vee (p\ (ap\ (ap\ (c.2Ebool.2EIN \\ A.27a)\ V2x)\ V1t)))))) \end{aligned} \quad (63)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\ (2^{A.27a}).((ap\ (ap\ (c.2Epred_set.2EUNION\ A.27a)\ V0s)\ V1t) = (\\ ap\ (ap\ (c.2Epred_set.2EUNION\ A.27a)\ V1t)\ V0s)))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\ (2^{A.27a}).(\forall V2x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a) \\ V2x)\ (ap\ (ap\ (c.2Epred_set.2EINTER\ A.27a)\ V0s)\ V1t)))) \Leftrightarrow ((p\ (ap \\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c.2Ebool.2EIN \\ A.27a)\ V2x)\ V1t)))))) \end{aligned} \quad (65)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\ (2^{A.27a}).((ap\ (ap\ (c.2Epred_set.2EINTER\ A.27a)\ V0s)\ V1t) = (\\ ap\ (ap\ (c.2Epred_set.2EINTER\ A.27a)\ V1t)\ V0s)))) \end{aligned} \quad (66)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in ty.2Erealx.2Ereal.(\forall V1y \in ty.2Erealx.2Ereal. \\ ((ap\ (ap\ c.2Erealx.2Ereal_add\ V0x)\ V1y) = (ap\ c.2Erealx.2Ereal_add \\ V1y)\ V0x)))) \end{aligned} \quad (67)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
V0x) (ap (ap c_2Erealax_2Ereal_add V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_add \\
& (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) V2z))))))
\end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
(ap c_2Erealax_2Ereal_neg V0x)) V0x) = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
((ap (ap c_2Erealax_2Ereal_mul V0x) V1y) = (ap (ap c_2Erealax_2Ereal_mul \\
& V1y) V0x))))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
(ap c_2Ereal_2Ereal_of_num (ap c_2Earithmic_2ENUMERAL (\\
& ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))) V0x) = V0x))
\end{aligned} \tag{71}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
V0x) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = V0x))
\end{aligned} \tag{72}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
V0x) (ap c_2Erealax_2Ereal_neg V0x)) = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))
\end{aligned} \tag{73}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
((ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_add V0x) \\
& V1y)) = (ap (ap c_2Erealax_2Ereal_add (ap c_2Erealax_2Ereal_neg \\
& V0x)) (ap c_2Erealax_2Ereal_neg V1y))))))
\end{aligned} \tag{74}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
(ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))
\end{aligned} \tag{75}$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(p (ap (ap c_2Ereal_2Ereal_lte V0x) V0x))) \quad (76)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.(\forall V2z \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Erealax_2Ereal_lt V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V1y) V2z))) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt V0x) V2z))))))) \quad (77)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.(\forall V2z \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lt V0x) V1y)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt V1y) V2z))) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt V0x) V2z))))))) \quad (78)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.(\forall V2z \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V1y) V2z))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte V0x) V2z))))))) \quad (79)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V1y) V0x)))) \Leftrightarrow (V0x = V1y)))) \quad (80)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.(((ap (ap c_2Ereal_2Ereal_sub V0x) V1y) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \Leftrightarrow (V0x = V1y)))) \quad (81)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V1y))) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) (ap (ap c_2Ereal_2E_2F V0x) V1y)))))) \quad (82)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap c_2Ereal_2Eabs (ap c_2Erealax_2Ereal_neg V0x)) = (ap c_2Ereal_2Eabs V0x))) \quad (83)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap\ c_2Ereal_2Eabs\ (ap\ (ap\ c_2Erealax_2Ereal_mul\ V0x)\ V1y)) = \\
& (ap\ (ap\ c_2Erealax_2Ereal_mul\ (ap\ c_2Ereal_2Eabs\ V0x))\ (ap\ c_2Ereal_2Eabs \\
& \quad V1y))))))
\end{aligned} \tag{84}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((\neg(V0x = (ap\ c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))) \Leftrightarrow (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0))\ (ap\ c_2Ereal_2Eabs\ V0x))))))
\end{aligned} \tag{85}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap\ (ap\ c_2Erealax_2Ereal_mul\ V0x)\ (ap\ c_2Erealax_2Ereal_neg \\
& \quad V1y)) = (ap\ c_2Erealax_2Ereal_neg\ (ap\ (ap\ c_2Erealax_2Ereal_mul \\
& \quad \quad V0x)\ V1y))))))
\end{aligned} \tag{86}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap\ (ap\ c_2Erealax_2Ereal_mul\ (ap\ c_2Erealax_2Ereal_neg\ V0x)) \\
& \quad V1y) = (ap\ c_2Erealax_2Ereal_neg\ (ap\ (ap\ c_2Erealax_2Ereal_mul \\
& \quad \quad V0x)\ V1y))))))
\end{aligned} \tag{87}$$

Assume the following.

$$\begin{aligned}
& (\forall V0y \in ty_2Erealax_2Ereal. (\forall V1x \in ty_2Erealax_2Ereal. \\
& ((p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ V1x)\ V0y)) \Leftrightarrow (\neg(p\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\
& \quad V0y)\ V1x))))))
\end{aligned} \tag{88}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ c_2Erealax_2Ereal_neg\ V0x)) \\
& \quad V1y)) \Leftrightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad \quad c_2Enum_2E0))\ (ap\ (ap\ c_2Erealax_2Ereal_add\ V0x)\ V1y))))))
\end{aligned} \tag{89}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ c_2Erealax_2Ereal_neg\ V0x)) \\
& \quad (ap\ c_2Erealax_2Ereal_neg\ V1y))) \Leftrightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\
& \quad \quad V1y)\ V0x))))))
\end{aligned} \tag{90}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap\ c_2Erealax_2Ereal_neg \\
& \quad (ap\ c_2Erealax_2Ereal_neg\ V0x)) = V0x))
\end{aligned} \tag{91}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte V0x) (ap c_2Erealax_2Ereal_neg \\
& V1y)))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap (ap c_2Erealax_2Ereal_add \\
& V0x) V1y)) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))))))
\end{aligned} \tag{92}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((p (ap (ap c_2Erealax_2Ereal_lt \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V2z)) \Rightarrow ((p (ap (ap \\
& c_2Erealax_2Ereal_lt V0x) (ap (ap c_2Ereal_2E2F V1y) V2z)))) \Leftrightarrow \\
& (p (ap (ap c_2Erealax_2Ereal_lt (ap (ap c_2Erealax_2Ereal_mul \\
& V0x) V2z)) V1y))))))
\end{aligned} \tag{93}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& V0x) (ap (ap c_2Erealax_2Ereal_add V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_add \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)) (ap (ap c_2Erealax_2Ereal_mul \\
& V0x) V2z))))))
\end{aligned} \tag{94}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) V2z) = (ap (ap c_2Erealax_2Ereal_add \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V2z)) (ap (ap c_2Erealax_2Ereal_mul \\
& V1y) V2z))))))
\end{aligned} \tag{95}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \forall A_27c. \\
& \text{nonempty } A_27c \Rightarrow \forall A_27d.\text{nonempty } A_27d \Rightarrow \forall A_27e.\text{nonempty } \\
& A_27e \Rightarrow \forall A_27f.\text{nonempty } A_27f \Rightarrow \forall A_27g.\text{nonempty } A_27g \Rightarrow \\
& (\forall V0Q \in (2^{A_27b}).((\forall V1P \in (2^{A_27a}).(\forall V2f \in \\
& (A_27b^{A_27a}).((\exists V3z \in A_27b.((p (ap (ap (c_2Ebool_2EIN \\
& A_27b) V3z) (ap (c_2Epred_set_2EGSPEC A_27b A_27a) (\lambda V4x \in \\
& A_27a.(ap (ap (c_2Epair_2E_2C A_27b 2) (ap V2f V4x)) (ap V1P V4x)))))) \wedge \\
& (p (ap V0Q V3z)))))) \Leftrightarrow (\exists V5x \in A_27a.((p (ap V1P V5x)) \wedge (p (ap V0Q \\
& (ap V2f V5x)))))) \wedge ((\forall V6P \in ((2^{A_27d})^{A_27c}).(\forall V7f \in \\
& ((A_27b^{A_27d})^{A_27c}).((\exists V8z \in A_27b.((p (ap (ap (c_2Ebool_2EIN \\
& A_27b) V8z) (ap (c_2Epred_set_2EGSPEC A_27b (ty_2Epair_2Eprod \\
& A_27c A_27d)) (ap (c_2Epair_2EUNCURRY A_27c A_27d (ty_2Epair_2Eprod \\
& A_27b 2)) (\lambda V9x \in A_27c.(\lambda V10y \in A_27d.(ap (ap (c_2Epair_2E_2C \\
& A_27b 2) (ap (ap V7f V9x) V10y)) (ap (ap V6P V9x) V10y)))))) \wedge (p \\
& (ap V0Q V8z)))))) \Leftrightarrow (\exists V11x \in A_27c.(\exists V12y \in A_27d.((p \\
& (ap (ap V6P V11x) V12y)) \wedge (p (ap V0Q (ap (ap V7f V11x) V12y)))))) \wedge \\
& (\forall V13P \in (((2^{A_27g})^{A_27f})^{A_27e}).(\forall V14f \in (((A_27b^{A_27g})^{A_27f})^{A_27e}). \\
& ((\exists V15z \in A_27b.((p (ap (ap (c_2Ebool_2EIN A_27b) V15z) (\\
& ap (c_2Epred_set_2EGSPEC A_27b (ty_2Epair_2Eprod A_27e (ty_2Epair_2Eprod \\
& A_27f A_27g))) (ap (c_2Epair_2EUNCURRY A_27e (ty_2Epair_2Eprod \\
& A_27f A_27g) (ty_2Epair_2Eprod A_27b 2)) (\lambda V16w \in A_27e.(ap \\
& (c_2Epair_2EUNCURRY A_27f A_27g (ty_2Epair_2Eprod A_27b 2)) \\
& (\lambda V17x \in A_27f.(\lambda V18y \in A_27g.(ap (ap (c_2Epair_2E_2C A_27b \\
& 2) (ap (ap (ap V14f V16w) V17x) V18y)) (ap (ap (ap V13P V16w) V17x) \\
& V18y)))))) \wedge (p (ap V0Q V15z)))))) \Leftrightarrow (\exists V19w \in A_27e.(\exists V20x \in \\
& A_27f.(\exists V21y \in A_27g.((p (ap (ap (ap V13P V19w) V20x) V21y)) \wedge \\
& (p (ap V0Q (ap (ap (ap V14f V19w) V20x) V21y)))))))))
\end{aligned} \tag{96}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in ((2^{ty_2Erealax_2Ereal})^{ty_2Erealax_2Ereal}). \\
& (((\forall V1x \in ty_2Erealax_2Ereal.(\forall V2y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap V0P V1x) V2y)) \Leftrightarrow (p (ap (ap V0P V2y) V1x)))))) \wedge (\forall V3x \in \\
& ty_2Erealax_2Ereal.(\forall V4y \in ty_2Erealax_2Ereal.((p (ap \\
& (ap c_2Ereal_2Ereal_lte V3x) V4y)) \Rightarrow (p (ap (ap V0P V3x) V4y)))))) \Rightarrow \\
& (\forall V5x \in ty_2Erealax_2Ereal.(\forall V6y \in ty_2Erealax_2Ereal. \\
& (p (ap (ap V0P V5x) V6y))))))
\end{aligned} \tag{97}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap c_2Ereal_topology_2EDist (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal) V0x) V1y)) = (ap c_2Ereal_2Eabs (ap (ap c_2Ereal_2Ereal_sub \\
& V0x) V1y))))))
\end{aligned} \tag{98}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((\neg(V0x = V1y)) \Leftrightarrow (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) (ap c_2Ereal_topology_2EDist (ap (ap (c_2Epair_2E_2C \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0x) V1y))))))
\end{aligned} \tag{99}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Erealax_2Ereal. (\forall V1b \in ty_2Erealax_2Ereal. \\
& ((\neg(V0a = V1b)) \Rightarrow ((ap c_2Ereal_topology_2Eopen_segment (ap \\
& (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
V0a) V1b)) = (ap (c_2Epred_set_2EGSPEC ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& (\lambda V2u \in ty_2Erealax_2Ereal. (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& 2) (ap (ap c_2Erealax_2Ereal_add (ap (ap c_2Erealax_2Ereal_mul \\
& (ap (ap c_2Ereal_2Ereal_sub (ap c_2Ereal_2Ereal_of_num (ap \\
& c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))) \\
V2u)) V0a)) (ap (ap c_2Erealax_2Ereal_mul V2u) V1b))) (ap (ap c_2Ebool_2E_2F_5C \\
& (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \\
& V2u)) (ap (ap c_2Erealax_2Ereal_lt V2u) (ap c_2Ereal_2Ereal_of_num \\
& (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO))))))))))
\end{aligned} \tag{100}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Erealax_2Ereal. (\forall V1b \in ty_2Erealax_2Ereal. \\
& (((ap\ c_2Ereal_topology_2Eclosed_segment\ (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)) \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal) \\
& \quad V0a\ V1b))\ (c_2Elist_2ENIL\ (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal \\
& \quad ty_2Erealax_2Ereal)))) = (ap\ (c_2Epred_set_2EGSPEC\ ty_2Erealax_2Ereal \\
& \quad ty_2Erealax_2Ereal)\ (\lambda V2u \in ty_2Erealax_2Ereal. (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad ty_2Erealax_2Ereal\ 2)\ (ap\ (ap\ c_2Erealax_2Ereal_add\ (ap\ (ap \\
& \quad c_2Erealax_2Ereal_mul\ (ap\ (ap\ c_2Ereal_2Ereal_sub\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))) \\
& \quad V2u))\ V0a))\ (ap\ (ap\ c_2Erealax_2Ereal_mul\ V2u\ V1b)))\ (ap\ (ap\ c_2Ebool_2E_2F_5C \\
& \quad (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) \\
& \quad V2u))\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ V2u)\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))))))) \wedge \\
& ((ap\ c_2Ereal_topology_2Eopen_segment\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ V0a\ V1b)) = (ap\ (ap\ (c_2Epred_set_2EDIFF \\
& \quad ty_2Erealax_2Ereal)\ (ap\ c_2Ereal_topology_2Eclosed_segment \\
& \quad (ap\ (ap\ (c_2Elist_2ECONS\ (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal \\
& \quad ty_2Erealax_2Ereal))\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal \\
& \quad ty_2Erealax_2Ereal)\ V0a\ V1b))\ (c_2Elist_2ENIL\ (ty_2Epair_2Eprod \\
& \quad ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal))))))\ (ap\ (ap\ (c_2Epred_set_2EINSERT \\
& \quad ty_2Erealax_2Ereal)\ V0a)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ ty_2Erealax_2Ereal \\
& \quad V1b)\ (c_2Epred_set_2EEMPTY\ ty_2Erealax_2Ereal))))))))) \\
& \hspace{15em} (101)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0a \in ty_2Erealax_2Ereal. ((ap\ c_2Ereal_topology_2Eclosed_segment \\
& \quad (ap\ (ap\ (c_2Elist_2ECONS\ (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal \\
& \quad ty_2Erealax_2Ereal))\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal \\
& \quad ty_2Erealax_2Ereal)\ V0a\ V0a))\ (c_2Elist_2ENIL\ (ty_2Epair_2Eprod \\
& \quad ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)))) = (ap\ (ap\ (c_2Epred_set_2EINSERT \\
& \quad ty_2Erealax_2Ereal)\ V0a)\ (c_2Epred_set_2EEMPTY\ ty_2Erealax_2Ereal)))) \wedge \\
& (\forall V1a \in ty_2Erealax_2Ereal. ((ap\ c_2Ereal_topology_2Eopen_segment \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal) \\
& \quad V1a\ V1a)) = (c_2Epred_set_2EEMPTY\ ty_2Erealax_2Ereal)))))) \\
& \hspace{15em} (102)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (p\ (ap\ c_2Ereal_topology_2Econnected\ (c_2Epred_set_2EEMPTY \\
& \quad ty_2Erealax_2Ereal))) \hspace{10em} (103)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Erealax_2Ereal. (p\ (ap\ c_2Ereal_topology_2Econnected \\
& \quad (ap\ (ap\ (c_2Epred_set_2EINSERT\ ty_2Erealax_2Ereal)\ V0a)\ (c_2Epred_set_2EEMPTY \\
& \quad ty_2Erealax_2Ereal)))))) \\
& \hspace{15em} (104)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1a \in \\
& ty_2Erealax_2Ereal.(\forall V2b \in ty_2Erealax_2Ereal.(\forall V3e1 \in \\
& (2^{ty_2Erealax_2Ereal}).(\forall V4e2 \in (2^{ty_2Erealax_2Ereal}). \\
& ((p (ap (ap (ap c_2Ereal_2Ereal_lte V1a) V2b)) \wedge ((p (ap (ap (c_2Ebool_2EIN \\
& ty_2Erealax_2Ereal) (ap V0f V1a)) V3e1)) \wedge ((p (ap (ap (c_2Ebool_2EIN \\
& ty_2Erealax_2Ereal) (ap V0f V2b)) V4e2)) \wedge ((\forall V5e \in ty_2Erealax_2Ereal. \\
& (\forall V6x \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lte \\
& V1a) V6x)) \wedge ((p (ap (ap c_2Ereal_2Ereal_lte V6x) V2b)) \wedge (ap (\\
& ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \\
& V5e)))))) \Rightarrow (\exists V7d \in ty_2Erealax_2Ereal.((p (ap (ap c_2Erealax_2Ereal_lt \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V7d)) \wedge (\forall V8y \in \\
& ty_2Erealax_2Ereal.((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Eabs \\
& (ap (ap c_2Ereal_2Ereal_sub V8y) V6x))) V7d)) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt \\
& (ap c_2Ereal_topology_2EDist (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal) (ap V0f V8y)) (ap V0f V6x)))) V5e)))))) \wedge \\
& ((\forall V9y \in ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN \\
& ty_2Erealax_2Ereal) V9y) V3e1)) \Rightarrow (\exists V10e \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V10e)) \wedge (\forall V11y_27 \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_topology_2EDist \\
& (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V11y_27) V9y))) V10e)) \Rightarrow (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V11y_27) V3e1)))))) \wedge ((\forall V12y \in ty_2Erealax_2Ereal. (\\
& (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V12y) V4e2)) \Rightarrow (\exists V13e \in \\
& ty_2Erealax_2Ereal.((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V13e)) \wedge (\forall V14y_27 \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_topology_2EDist \\
& (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V14y_27) V12y))) V13e)) \Rightarrow (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V14y_27) V4e2)))))) \wedge (\neg (\exists V15x \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte V1a) V15x)) \wedge ((p (ap (ap c_2Ereal_2Ereal_lte \\
& V15x) V2b)) \wedge ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap \\
& V0f V15x)) V3e1)) \wedge (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& (ap V0f V15x)) V4e2)))))) \Rightarrow (\exists V16x \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte V1a) V16x)) \wedge ((p (ap (ap c_2Ereal_2Ereal_lte \\
& V16x) V2b)) \wedge (\neg (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& (ap V0f V16x)) V3e1))) \wedge (\neg (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& (ap V0f V16x)) V4e2)))))))))
\end{aligned} \tag{105}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{106}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{107}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (108)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (109)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (110)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (111)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (112)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (113)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (114)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (115)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (116)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (117)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (118)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (119)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (120)$$

Theorem 1

$$\begin{aligned} & ((\forall V0a \in ty_2Erealax_2Ereal.(\forall V1b \in ty_2Erealax_2Ereal. \\ & (p (ap \ c_2Ereal_topology_2Econnected (ap \ c_2Ereal_topology_2Eclosed_segment \\ & (ap (ap (c_2Elist_2ECONS (ty_2Epair_2Eprod ty_2Erealax_2Ereal \\ & ty_2Erealax_2Ereal)) (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\ & ty_2Erealax_2Ereal) V0a) V1b)) (c_2Elist_2ENIL (ty_2Epair_2Eprod \\ & ty_2Erealax_2Ereal ty_2Erealax_2Ereal)))))) \wedge (\forall V2a \in \\ & ty_2Erealax_2Ereal.(\forall V3b \in ty_2Erealax_2Ereal.(p (ap \\ & c_2Ereal_topology_2Econnected (ap \ c_2Ereal_topology_2Eopen_segment \\ & (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\ & V2a) V3b))))))))) \end{aligned}$$