thm_2Ereal__topology_2ECONTINUOUS__AT__INV (TMGJE2QCbk73y6zufMZk1QxMCZKpEFRc3T7)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Epred__set_2EUNIV *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2ET})$.

Let $ty\_2Ereal\_\_topology\_2Enet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ereal\_\_topology\_2Enet\ A0) \tag{1}$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{2}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{3}$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{4}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{5}$$

Definition 6 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 7 *We define* $\mathsf{c\_2Erealax\_2Ereal\_\_REP}$ *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (\mathsf{c\_2Emin\_2E\_40}\ (ty$

Let $c\_2Erealax\_2Etreal\_\_inv : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{6}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{7}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr} \tag{8}$$

Definition 8 *We define* $\mathsf{c\_2Erealax\_2Ereal\_\_ABS}$ *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_$

Definition 9 *We define* $\mathsf{c\_2Erealax\_2Einv}$ *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ \mathsf{c\_2Erealax\_2Ereal\_\_ABS}\ ($

Definition 10 *We define* $\mathsf{c\_2Ecombin\_2Eo}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{9}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{10}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{11}$$

Definition 11 *We define* $\mathsf{c\_2Enum\_2E0}$ *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{12}$$

Definition 12 *We define* $\mathsf{c\_2Emin\_2E\_3D\_3D\_3E}$ *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 13 *We define* $\mathsf{c\_2Ebool\_2E\_7E}$ *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 14 *We define* $\mathsf{c\_2Ebool\_2E\_2F\_5C}$ *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{13}$$

Definition 15 *We define* $\mathsf{c\_2Epair\_2E\_2C}$ *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0x\in A\_27a.\lambda V1y\in A\_27b.(ap\ (c\_2$

Let $c\_2Ereal\_\_topology\_2EDist:\iota$ be given. Assume the following.

$$c\_2Ereal\_\_topology\_2EDist\in(ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \tag{14}$$

Let $c\_2Erealax\_2Etreal\_\_lt:\iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt\in((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{15}$$

Definition 16 *We define* $\mathsf{c\_2Erealax\_2Ereal\_\_lt}$ *to be* $\lambda V0T1\in ty\_2Erealax\_2Ereal.\lambda V1T2\in ty\_2Erealax\_$

Definition 17 *We define* $\mathsf{c\_2Ereal\_2Ereal\_\_lte}$ *to be* $\lambda V0x\in ty\_2Erealax\_2Ereal.\lambda V1y\in ty\_2Erealax\_2Er$

Let $c\_2Ereal\_\_topology\_2Emk\_\_net:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Ereal\_\_topology\_2Emk\_\_net\ A\_27a\in((ty\_2Ereal\_\_topology\_2Enet\ A\_27a)^{((2^{A\_27a})^{A\_27a})}) \tag{16}$$

Definition 18 *We define* $\mathsf{c\_2Ereal\_\_topology\_2Eat}$ *to be* $\lambda V0a\in ty\_2Erealax\_2Ereal.(ap\ (c\_2Ereal\_\_topolo$

Definition 19 *We define* $\mathsf{c\_2Ebool\_2EIN}$ *to be* $\lambda A\_27a:\iota.(\lambda V0x\in A\_27a.(\lambda V1f\in(2^{A\_27a}).(ap\ V1f\ V0x))$

Let $c\_2Ereal\_\_topology\_2Enetord:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Ereal\_\_topology\_2Enetord\ A\_27a\in(((2^{A\_27a})^{A\_27a})^{(ty\_2Ereal\_\_topology\_2Enet\ A\_27a)}) \tag{17}$$

Definition 20 *We define* $\mathsf{c\_2Ereal\_\_topology\_2Ewithin}$ *to be* $\lambda A\_27a:\iota.\lambda V0net\in(ty\_2Ereal\_\_topology\_2En$

Definition 21 *We define* $\mathsf{c\_2Ereal\_\_topology\_2Enetlimit}$ *to be* $\lambda A\_27a:\iota.\lambda V0net\in(ty\_2Ereal\_\_topology\_2E$

Definition 22 *We define* $\mathsf{c\_2Ebool\_2E\_3F}$ *to be* $\lambda A\_27a:\iota.(\lambda V0P\in(2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 23 *We define* $\mathsf{c\_2Ebool\_2E\_5C\_2F}$ *to be* $(\lambda V0t1\in 2.(\lambda V1t2\in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t\in$

Definition 24 *We define* $\mathsf{c\_2Ereal\_\_topology\_2Etrivial\_\_limit}$ *to be* $\lambda A\_27a:\iota.\lambda V0net\in(ty\_2Ereal\_\_topology$

Definition 25 *We define* $\mathsf{c\_2Ereal\_\_topology\_2Eeventually}$ *to be* $\lambda A\_27a:\iota.\lambda V0p\in(2^{A\_27a}).\lambda V1net\in(ty\_2$

Definition 26 *We define* $\mathsf{c\_2Ereal\_\_topology\_2E\_2D\_2D\_3E}$ *to be* $\lambda A\_27a:\iota.\lambda V0f\in(ty\_2Erealax\_2Ereal^{A}$

Definition 27 *We define* $\mathsf{c\_2Ereal\_\_topology\_2Econtinuous}$ *to be* $\lambda A\_27a:\iota.\lambda V0f\in(ty\_2Erealax\_2Ereal^{A}$

Assume the following.

$$True \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t\in 2.((\forall V1x\in A\_27a.(p\ V0t))\Leftrightarrow(p\ V0t))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{21}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Ewithin}\ ty\_2Erealax\_2Ereal)\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Eat}\ V0x))\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ ty\_2Erealax\_2Ereal)) = (ap\ \mathsf{c\_2Ereal\_\_topology\_2Eat}\ V0x))) \tag{22}$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V2a \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Econtinuous}\ ty\_2Erealax\_2Ereal)\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Ewithin}\ ty\_2Erealax\_2Ereal)\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Eat}\ V2a))\ V1s))) \wedge (\neg ((ap\ V0f\ V2a) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Econtinuous}\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Erealax\_2Einv})\ V0f))\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Ewithin}\ ty\_2Erealax\_2Ereal)\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Eat}\ V2a))\ V1s))))))) \tag{23}$$

Theorem 1

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1a \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Econtinuous}\ ty\_2Erealax\_2Ereal)\ V0f)\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Eat}\ V1a))) \wedge (\neg ((ap\ V0f\ V1a) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Econtinuous}\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Erealax\_2Einv})\ V0f))\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Eat}\ V1a))))))$$