thm_2Ereal__topology_2ECONTINUOUS__AT__WITHIN__INV (TMdw2PmWF7arn5iZ9QTqozVjkurQPAQuZQ6)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{2}$$

Definition 7 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Let $c\_2Ereal\_\_topology\_2EDist : \iota$ be given. Assume the following.

$$c\_2Ereal\_\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \tag{4}$$

Let $ty\_2Ereal\_\_topology\_2Enet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ereal\_\_topology\_2Enet\ A0) \tag{5}$$

Let $c\_2Ereal\_\_topology\_2Enetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ereal\_\_topology\_2Enetord\ A\_27a \in (((2^{A\_27a})^{A\_27a})^{(ty\_2Ereal\_\_topology\_2Enet\ A\_27a)}) \tag{6}$$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 9 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40 A

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{7}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{8}$$

Definition 10 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(t$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehr} \tag{9}$$

Definition 11 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Definition 12 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 13 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 14 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{10}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{11}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{12}$$

Definition 15 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{13}$$

Let $c\_2Ereal\_\_topology\_2Emk\_\_net : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ereal\_\_topology\_2Emk\_\_net\ A\_27a \in ((ty\_2Ereal\_\_topology\_2Enet\ A\_27a)^{((2^{A\_27a})^{A\_27a})}) \tag{14}$$

Definition 16 *We define* c_2Ereal__topology_2Eat *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Ereal\_\_topolo$

Definition 17 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 18 *We define* c_2Ereal__topology_2Ewithin *to be* $\lambda A\_27a : \iota.\lambda V0net \in (ty\_2Ereal\_\_topology\_2En$

Definition 19 *We define* c_2Ereal__topology_2Etrivial__limit *to be* $\lambda A\_27a : \iota.\lambda V0net \in (ty\_2Ereal\_\_topology$

Definition 20 *We define* c_2Ereal__topology_2Eeventually *to be* $\lambda A\_27a : \iota.\lambda V0p \in (2^{A\_27a}).\lambda V1net \in (ty\_2$

Definition 21 *We define* c_2Ereal__topology_2E_2D_2D_3E *to be* $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Erealax\_2Ereal^{A}$

Let $c\_2Erealax\_2Etreal\_\_inv : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{15}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{16}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr} \tag{17}$$

Definition 22 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty$

Definition 23 *We define* c_2Erealax_2Einv *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal__ABS

Definition 24 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1$

Definition 25 *We define* c_2Ereal__topology_2Enetlimit *to be* $\lambda A\_27a : \iota.\lambda V0net \in (ty\_2Ereal\_\_topology\_2E$

Definition 26 *We define* c_2Ereal__topology_2Econtinuous *to be* $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Erealax\_2Ereal^{A}$

Assume the following.

$$True \tag{18}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{19}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg (p\ V0t)))) \tag{21}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{24}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{26}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{27}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{28}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{29}$$

Assume the following.

$$\forall \mathit{A\_27a}.\mathit{nonempty}\ \mathit{A\_27a}{\Rightarrow}(\forall \mathit{V0f} \in (\mathit{ty\_2Erealax\_2Ereal}^{\mathit{A\_27a}}).(\forall \mathit{V1l} \in \mathit{ty\_2Erealax\_2Ereal}.(\forall \mathit{V2net} \in (\mathit{ty\_2Ereal\_\_topology\_2Enet}\ \mathit{A\_27a}).((p\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Ereal\_\_topology\_2E\_2D\_2D\_3E}\ \mathit{A\_27a})\ \mathit{V0f})\ \mathit{V1l})\ \mathit{V2net})){\Leftrightarrow}((p\ (\mathit{ap}\ (\mathsf{c\_2Ereal\_\_topology\_2Etrivial\_\_limit}\ \mathit{A\_27a})\ \mathit{V2net})){\vee}(\forall \mathit{V3e} \in \mathit{ty\_2Erealax\_2Ereal}.((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (\mathit{ap}\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ \mathit{V3e})){\Rightarrow}(\exists \mathit{V4y} \in \mathit{A\_27a}.((\exists \mathit{V5x} \in \mathit{A\_27a}.(p\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2Ereal\_\_topology\_2Enetord}\ \mathit{A\_27a})\ \mathit{V2net})\ \mathit{V5x})\ \mathit{V4y}))){\wedge}(\forall \mathit{V6x} \in \mathit{A\_27a}.((p\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2Ereal\_\_topology\_2Enetord}\ \mathit{A\_27a})\ \mathit{V2net})\ \mathit{V6x})\ \mathit{V4y})){\Rightarrow}(p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (\mathit{ap}\ \mathit{c\_2Ereal\_\_topology\_2EDist}\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Epair\_2E\_2C}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal})\ (\mathit{ap}\ \mathit{V0f}\ \mathit{V6x}))\ \mathit{V1l})))\ \mathit{V3e})))))))))))))) \tag{30}$$

Assume the following.

$$(\forall \mathit{V0a} \in \mathit{ty\_2Erealax\_2Ereal}.(\forall \mathit{V1s} \in (2^{\mathit{ty\_2Erealax\_2Ereal}}).((\neg(p\ (\mathit{ap}\ (\mathsf{c\_2Ereal\_\_topology\_2Etrivial\_\_limit}\ \mathit{ty\_2Erealax\_2Ereal})\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Ereal\_\_topology\_2Ewithin}\ \mathit{ty\_2Erealax\_2Ereal})\ (\mathit{ap}\ \mathsf{c\_2Ereal\_\_topology\_2Eat}\ \mathit{V0a}))\ \mathit{V1s})))){\Rightarrow}((\mathit{ap}\ (\mathsf{c\_2Ereal\_\_topology\_2Enetlimit}\ \mathit{ty\_2Erealax\_2Ereal})\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Ereal\_\_topology\_2Ewithin}\ \mathit{ty\_2Erealax\_2Ereal})\ (\mathit{ap}\ \mathsf{c\_2Ereal\_\_topology\_2Eat}\ \mathit{V0a}))\ \mathit{V1s})) = \mathit{V0a})))) \tag{31}$$

Assume the following.

$$\forall \mathit{A\_27a}.\mathit{nonempty}\ \mathit{A\_27a}{\Rightarrow}(\forall \mathit{V0net} \in (\mathit{ty\_2Ereal\_\_topology\_2Enet}\ \mathit{A\_27a}).(\forall \mathit{V1f} \in (\mathit{ty\_2Erealax\_2Ereal}^{\mathit{A\_27a}}).(((p\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Ereal\_\_topology\_2Econtinuous}\ \mathit{A\_27a})\ \mathit{V1f})\ \mathit{V0net})){\wedge}(\neg((\mathit{ap}\ \mathit{V1f}\ (\mathit{ap}\ (\mathsf{c\_2Ereal\_\_topology\_2Enetlimit}\ \mathit{A\_27a})\ \mathit{V0net})) = (\mathit{ap}\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0})))){\Rightarrow}(p\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Ereal\_\_topology\_2Econtinuous}\ \mathit{A\_27a})\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Ecombin\_2Eo}\ \mathit{A\_27a}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal})\ \mathsf{c\_2Erealax\_2Einv})\ \mathit{V1f}))\ \mathit{V0net}))))) \tag{32}$$

Theorem 1

$$(\forall \mathit{V0f} \in (\mathit{ty\_2Erealax\_2Ereal}^{\mathit{ty\_2Erealax\_2Ereal}}).(\forall \mathit{V1s} \in (2^{\mathit{ty\_2Erealax\_2Ereal}}).(\forall \mathit{V2a} \in \mathit{ty\_2Erealax\_2Ereal}.(((p\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Ereal\_\_topology\_2Econtinuous}\ \mathit{ty\_2Erealax\_2Ereal})\ \mathit{V0f})\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Ereal\_\_topology\_2Ewithin}\ \mathit{ty\_2Erealax\_2Ereal})\ (\mathit{ap}\ \mathsf{c\_2Ereal\_\_topology\_2Eat}\ \mathit{V2a}))\ \mathit{V1s}))){\wedge}(\neg((\mathit{ap}\ \mathit{V0f}\ \mathit{V2a}) = (\mathit{ap}\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0})))){\Rightarrow}(p\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Ereal\_\_topology\_2Econtinuous}\ \mathit{ty\_2Erealax\_2Ereal})\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Ecombin\_2Eo}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal})\ \mathsf{c\_2Erealax\_2Einv})\ \mathit{V0f}))\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Ereal\_\_topology\_2Ewithin}\ \mathit{ty\_2Erealax\_2Ereal})\ (\mathit{ap}\ \mathsf{c\_2Ereal\_\_topology\_2Eat}\ \mathit{V2a}))\ \mathit{V1s}))))))$$