

thm_2Ereal__topology_2ECONTINUOUS__CLOSED__PREIMAGE
(TMQ74gK98wYGi7esXBb9oNVTxpGGA6BcDxu)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_2E21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2EF$ to be $(ap (c_2Ebool_2E_2E21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2E7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2EF$

Definition 7 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E_2ET)$.

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Definition 8 We define $c_2Ebool_2E_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_2E21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \tag{3}$$

Definition 9 We define $c_2Epair_2E_2E2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (4)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (5)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (6)$$

Definition 10 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p\ (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p\ x)$ of type $\iota \Rightarrow \iota$).

Definition 11 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ (ap\ p\ x)))$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (7)$$

Definition 12 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 13 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \quad (8)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (10)$$

Definition 14 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (11)$$

Let $ty_2Ereal_topology_2Enet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ereal_topology_2Enet\ A0) \quad (12)$$

Let $c_2Ereal_topology_2Emk_net : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Ereal_topology_2Emk_net\ A.27a \in ((ty_2Ereal_topology_2Enet\ A.27a)^{(2^{A.27a})^{A.27a}}) \quad (13)$$

Definition 15 We define $c_Ereal_topology_2Eat$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Ereal_topology_2E$
Let $c_2Ereal_topology_2Eenetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ereal_topology_2Eenetord A_27a \in ((2^{A_27a})^{A_27a})^{(ty_2Ereal_topology_2Eenet A_27a)} \quad (14)$$

Definition 16 We define $c_2Ereal_topology_2Eenetlimit$ to be $\lambda A_27a : \iota.\lambda V0net \in (ty_2Ereal_topology_2E$

Definition 17 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 18 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 19 We define $c_2Ereal_topology_2Etrivial_limit$ to be $\lambda A_27a : \iota.\lambda V0net \in (ty_2Ereal_topology_2E$

Definition 20 We define $c_2Ereal_topology_2Eeventually$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1net \in (ty_2$

Definition 21 We define $c_2Ereal_topology_2E_2D_2D_3E$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal^A$

Definition 22 We define $c_2Ereal_topology_2Econtinuous$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal^A$

Definition 23 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (15)$$

Definition 24 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2$

Definition 25 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Ebool_2E_2$

Definition 26 We define $c_2Ereal_topology_2EClosed$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap c_2Ereal_topo$

Definition 27 We define $c_2Ereal_topology_2Econtinuous_on$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Ereal$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\
& (p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{20}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p V0t))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\
& ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\
& 2. (((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow \\
& (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (p (ap (ap (c_2Ebool_2EIN A_27a) V0x) (c_2Epred_set_2EUNIV A_27a)))) \tag{24}$$

Assume the following.

$$(p (ap c_2Ereal_topology_2EClosed (c_2Epred_set_2EUNIV ty_2Erealax_2Ereal))) \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V1s \in \\
& (2^{ty_2Erealax_2Ereal}). ((\forall V2x \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V2x) V1s)) \Rightarrow (p (\\
& ap (ap (c_2Ereal_topology_2Econtinuous ty_2Erealax_2Ereal) \\
& V0f) (ap c_2Ereal_topology_2Eat V2x)))))) \Rightarrow (p (ap (ap c_2Ereal_topology_2Econtinuous_on \\
& V0f) V1s))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in \\
& \quad (2^{ty_2Erealax_2Ereal}).(\forall V2t \in (2^{ty_2Erealax_2Ereal}). \\
& \quad ((p (ap (ap c_2Ereal_topology_2Econtinuous_on V0f) V1s)) \wedge \\
& \quad ((p (ap c_2Ereal_topology_2EClosed V1s)) \wedge (p (ap c_2Ereal_topology_2EClosed \\
& \quad V2t)))) \Rightarrow (p (ap c_2Ereal_topology_2EClosed (ap (c_2Epred_set_2EGSPEC \\
& \quad ty_2Erealax_2Ereal ty_2Erealax_2Ereal) (\lambda V3x \in ty_2Erealax_2Ereal. \\
& \quad (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal 2) V3x) (ap (ap c_2Ebool_2E_2F_5C \\
& \quad (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V3x) V1s)) (ap (ap (\\
& \quad c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap V0f V3x)) V2t))))))))))
\end{aligned} \tag{27}$$

Theorem 1

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in \\
& \quad (2^{ty_2Erealax_2Ereal}).((\forall V2x \in ty_2Erealax_2Ereal. \\
& \quad (p (ap (ap (c_2Ereal_topology_2Econtinuous ty_2Erealax_2Ereal) \\
& \quad V0f) (ap c_2Ereal_topology_2Eat V2x)))) \wedge (p (ap c_2Ereal_topology_2EClosed \\
& \quad V1s)))) \Rightarrow (p (ap c_2Ereal_topology_2EClosed (ap (c_2Epred_set_2EGSPEC \\
& \quad ty_2Erealax_2Ereal ty_2Erealax_2Ereal) (\lambda V3x \in ty_2Erealax_2Ereal. \\
& \quad (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal 2) V3x) (ap (ap (c_2Ebool_2EIN \\
& \quad ty_2Erealax_2Ereal) (ap V0f V3x)) V1s))))))))))
\end{aligned}$$