

# thm\_2Ereal\_\_topology\_2ECONTINUOUS\_\_COMPONENT\_\_COMP (TMb2WhnotB8E53UmkPGPUdN1fNa5ZL6HC3f)

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Let  $ty\_2Ereal\_topology\_2Eenet : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ereal\_topology\_2Eenet\ A0) \quad (1)$$

Let  $c\_2Ereal\_topology\_2Eenetord : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ereal\_topology\_2Eenetord\ A\_27a \in \left( (2^{A\_27a})^{A\_27a} \right)_{(ty\_2Ereal\_topology\_2Eenet\ A\_27a)} \quad (2)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27a}))\ (2^{A\_27a}))\ P))$

**Definition 4** We define  $c\_2Ebool\_2E\_EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_EF))$

**Definition 7** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if\ (\exists x \in A.p\ (ap\ P\ x))\ then\ (the\ (\lambda x.x \in A \wedge p))$  of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define  $c\_2Ereal\_topology\_2Eenetlimit$  to be  $\lambda A\_27a : \iota.\lambda V0net \in (ty\_2Ereal\_topology\_2Eenet\ A\_27a)$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (3)$$

**Definition 9** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t))\ t1))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (4)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (5)$$

**Definition 10** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b)\ x\ y)$

Let  $c\_2Ereal\_topology\_2EDist : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \quad (6)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (7)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (8)$$

**Definition 11** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E\_40\ ty\_2Erealax\_2Ereal)\ a)$

Let  $c\_2Erealax\_2Etreallt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreallt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (9)$$

**Definition 12** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.(c\_2Emin\_2E\_40\ T1\ T2)$

**Definition 13** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ ty\_2Erealax\_2Ereal)\ P)))$

**Definition 14** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ t1\ t2))\ (\lambda V2t \in 2.(ap\ (c\_2Ebool\_2E\_3F\ ty\_2Erealax\_2Ereal)\ t)))$

**Definition 15** We define  $c\_2Ereal\_topology\_2Etrivial\_limit$  to be  $\lambda A\_27a : \iota.\lambda V0net \in (ty\_2Ereal\_topology\_2Etrivial\_limit\ A\_27a)$

**Definition 16** We define  $c\_2Ereal\_topology\_2Eeventually$  to be  $\lambda A\_27a : \iota.\lambda V0p \in (2^{A\_27a}).\lambda V1net \in (ty\_2Ereal\_topology\_2Etrivial\_limit\ A\_27a).(ap\ V0p\ net)$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in omega \quad (10)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (11)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (12)$$



Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (19)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (22)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (23)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (25)$$

Assume the following.

$$(\forall V0r \in 2.(\forall V1p \in 2.(\forall V2q \in 2.(((p V1p) \wedge (p V2q)) \Rightarrow (p V0r)) \Leftrightarrow ((p V2q) \Rightarrow ((p V1p) \Rightarrow (p V0r)))))) \quad (26)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap (c\_2Ecombin\_2EI A\_27a) V0x) = V0x)) \quad (27)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(((ap (ap (c\_2Ecombin\_2Eo A\_27a A\_27b A\_27b) (c\_2Ecombin\_2EI A\_27b)) V0f) = V0f) \wedge ((ap (ap (c\_2Ecombin\_2Eo A\_27a A\_27b A\_27a) V0f) (c\_2Ecombin\_2EI A\_27a)) = V0f))) \quad (28)$$

Assume the following.

$$(p (ap (c.2Ereal\_topology\_2Elinear (\lambda V0x \in ty\_2Erealax\_2Ereal. V0x))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (32)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (33)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (34)$$

**Theorem 1**

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow ( \\ & \quad \forall V0net \in (ty\_2Ereal\_topology\_2Enet A.27a).(\forall V1f \in \\ & (ty\_2Erealax\_2Ereal^{A.27a}).(\forall V2i \in A.27b.((p (ap (ap (c.2Ereal\_topology\_2Econtinuous \\ & A.27a) V1f) V0net)) \Rightarrow (p (ap (ap (c.2Ereal\_topology\_2Econtinuous \\ & A.27a) (\lambda V3x \in A.27a.(ap V1f V3x))) V0net)))))) \end{aligned}$$