

thm_2Ereal__topology_2ECONTINUOUS__DISCONNECTED__RA
(TMSawj8n5PeX1iY48TXbve8ubjun6rL6rDt)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 4 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 6 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (2)$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (3)$$

Definition 9 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap (c_2E$

Definition 10 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 11 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Definition 12 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap (c_2Ebool_2E_21 2)$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (4)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (5)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (6)$$

Definition 13 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. p (ap P x)) \mathbf{then} (the (\lambda x. x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 14 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal. (ap (c_2Emin_2E_40 (the (\lambda x. x \in A \wedge P x))))$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal}) \quad (7)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal}) \quad (8)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (9)$$

Definition 15 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 16 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. (ap c_2Erealax_2Ereal_neg)$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal})^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal}) \quad (10)$$

Definition 17 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal. (ap c_2Erealax_2Ereal_add)$

Definition 18 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{11}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{12}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{13}$$

Definition 19 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \tag{14}$$

Let $c_2Erealax_2Etrealt_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealt_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \tag{15}$$

Definition 20 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 21 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E7E))$

Definition 22 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 23 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap\ (c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E7E))))$

Definition 24 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND))))$

Definition 25 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_3D_3D_3E\ V0P))))$

Definition 26 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Emin_2E_3D_3D_3E\ V0s\ V1t))$

Definition 27 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Emin_2E_3D_3D_3E\ V0s\ V1t))$

Definition 28 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Emin_2E_3D_3D_3E\ V0s\ V1t))$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \tag{16}$$

Definition 29 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (c_2Ebool_2E3F\ V0s))$

Definition 30 We define $c_2Ereal_topology_2Econnected$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ c_2Ebool_2E3F\ V0s)$

Definition 31 We define $c_2Ereal_topology_2Econnected_component$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).\lambda V$

Definition 32 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 33 We define $c_2Ereal_topology_2Econtinuous_on$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Ereal})$

Assume the following.

$$True \tag{17}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \tag{18}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{19}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0s \in (2^{ty_2Erealax_2Ereal}).((p (ap c_2Ereal_topology_2Econnected \\
& \quad V0s)) \Leftrightarrow (\forall V1f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). \\
& (\forall V2t \in (2^{ty_2Erealax_2Ereal}).(((p (ap (ap c_2Ereal_topology_2Econtinuous_on \\
& \quad V1f) V0s)) \wedge ((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) \\
& \quad (ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& \quad \quad V1f) V0s)) V2t)) \wedge (\forall V3y \in ty_2Erealax_2Ereal.((p (ap (ap \\
& (c_2Ebool_2EIN ty_2Erealax_2Ereal) V3y) V2t)) \Rightarrow ((ap (ap c_2Ereal_topology_2Econnected_component \\
& \quad V2t) V3y) = (ap (ap (c_2Epred_set_2EINSERT ty_2Erealax_2Ereal) \\
& \quad V3y) (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal)))))) \Rightarrow (\exists V4a \in \\
& \quad ty_2Erealax_2Ereal.(\forall V5x \in ty_2Erealax_2Ereal.((p (ap (ap \\
& (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V5x) V0s)) \Rightarrow ((ap V1f V5x) = \\
& \quad V4a)))))) \wedge ((\forall V6s \in (2^{ty_2Erealax_2Ereal}).((p (ap \\
& c_2Ereal_topology_2Econnected V6s)) \Leftrightarrow (\forall V7f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). \\
& \quad ((p (ap (ap c_2Ereal_topology_2Econtinuous_on V7f) V6s)) \wedge \\
& (\forall V8x \in ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& \quad V8x) V6s)) \Rightarrow (\exists V9e \in ty_2Erealax_2Ereal.((p (ap (ap c_2Erealax_2Ereal_lt \\
& \quad (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0) V9e)) \wedge (\forall V10y \in \\
& \quad ty_2Erealax_2Ereal.(((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V10y) V6s)) \wedge \neg((ap V7f V10y) = (ap V7f V8x)))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& \quad V9e) (ap c_2Ereal_2Eabs (ap (ap c_2Ereal_2Ereal_sub (ap V7f V10y)) \\
& \quad (ap V7f V8x)))))))))) \Rightarrow (\exists V11a \in ty_2Erealax_2Ereal.(\\
& \quad \forall V12x \in ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& \quad V12x) V6s)) \Rightarrow ((ap V7f V12x) = V11a)))))) \wedge (\forall V13s \in (2^{ty_2Erealax_2Ereal}). \\
& \quad ((p (ap c_2Ereal_topology_2Econnected V13s)) \Leftrightarrow (\forall V14f \in \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(((p (ap (ap c_2Ereal_topology_2Econtinuous_on \\
& \quad V14f) V13s)) \wedge (p (ap (c_2Epred_set_2EFINITE ty_2Erealax_2Ereal) \\
& \quad (ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& \quad \quad V14f) V13s)))) \Rightarrow (\exists V15a \in ty_2Erealax_2Ereal.(\forall V16x \in \\
& \quad ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& \quad V16x) V13s)) \Rightarrow ((ap V14f V16x) = V15a)))))))))) \\
& \hspace{15em} (20)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& ((\forall V0s \in (2^{ty_2Erealax_2Ereal}).((p (ap c_2Ereal_topology_2Econnected \\
& \quad V0s)) \Leftrightarrow (\forall V1f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). \\
& (\forall V2t \in (2^{ty_2Erealax_2Ereal}).(((p (ap (ap c_2Ereal_topology_2Econtinuous_on \\
& \quad V1f) V0s)) \wedge ((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) \\
& \quad (ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& \quad \quad V1f) V0s)) V2t)) \wedge (\forall V3y \in ty_2Erealax_2Ereal.((p (ap (ap \\
& (c_2Ebool_2EIN ty_2Erealax_2Ereal) V3y) V2t)) \Rightarrow ((ap (ap c_2Ereal_topology_2Econnected_component \\
& \quad V2t) V3y) = (ap (ap (c_2Epred_set_2EINSERT ty_2Erealax_2Ereal) \\
& \quad V3y) (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal)))))) \Rightarrow (\exists V4a \in \\
& \quad ty_2Erealax_2Ereal.(\forall V5x \in ty_2Erealax_2Ereal.((p (ap (ap \\
& (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V5x) V0s)) \Rightarrow ((ap V1f V5x) = \\
& \quad V4a))))))
\end{aligned}$$