

thm_2Ereal__topology_2ECONTINUOUS__GE__ON__CLOSURE (TMbM39Kw8YCjWfr1X5sAHKDNk36xHCLumzN)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2E$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2EF$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2EF$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 8 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty ty_2Ehreal_2Ehreal \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) ty_2Erealax_2Ereal) \tag{4}$$

Definition 9 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E40 (ty_2Erealax_2Ereal_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (5)$$

Definition 10 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 11 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 12 We define $c_2Ereal_2Ereal_ge$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 13 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E21\ 2) (\lambda V2t \in 2$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (6)$$

Definition 14 We define c_2Epair_2E2C to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (7)$$

Definition 15 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x))$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (8)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (9)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (11)$$

Definition 16 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (12)$$

Definition 17 We define $c_Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Ebool_2E2$

Definition 18 We define $c_Ereal_topology_2Elimit_point_of$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1s \in ($

Definition 19 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E21 2) (\lambda V2t \in$

Definition 20 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c$

Definition 21 We define $c_Ereal_topology_2Eclosure$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (ap (c_2Epred$

Definition 22 We define $c_Ereal_topology_2Econtinuous_on$ to be $\lambda V0f \in (ty_2Erealax_2Ereal)^{ty_2Ereal}$

Definition 23 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 24 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2$

Definition 25 We define $c_Ereal_topology_2EClosed$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap c_2Ereal_topo$

Assume the following.

$$True \tag{13}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{14}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{15}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \tag{17}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{18}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p V0t)))))) \tag{19}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (20)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))))) \Rightarrow 2.(((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \quad (21)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0P \in (2^{A_{27a}}).(\forall V1a \in A_{27a}.((\exists V2x \in A_{27a}.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (22)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow (\forall V0x \in A_{27a}.(\forall V1y \in A_{27b}.(\forall V2a \in A_{27a}.(\forall V3b \in A_{27b}.(((ap (ap (c_{2E}pair_{2E}2C A_{27a} A_{27b}) V0x) V1y) = (ap (ap (c_{2E}pair_{2E}2C A_{27a} A_{27b}) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \quad (23)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow (\forall V0f \in ((ty_{2E}pair_{2E}prod A_{27a} 2)^{A_{27b}}).(\forall V1v \in A_{27a}.((p (ap (ap (c_{2E}bool_{2E}IN A_{27a}) V1v) (ap (c_{2E}pred_{2E}set_{2E}EGSPEC A_{27a} A_{27b}) V0f))) \Leftrightarrow (\exists V2x \in A_{27b}.((ap (ap (c_{2E}pair_{2E}2C A_{27a} 2) V1v) c_{2E}bool_{2E}ET) = (ap V0f V2x)))))) \quad (24)$$

Assume the following.

$$(\forall V0a \in ty_{2E}realax_{2E}real.(p (ap c_{2E}real_{2E}topology_{2E}closed (ap (c_{2E}pred_{2E}set_{2E}EGSPEC ty_{2E}realax_{2E}real ty_{2E}realax_{2E}real) (\lambda V1x \in ty_{2E}realax_{2E}real.(ap (ap (c_{2E}pair_{2E}2C ty_{2E}realax_{2E}real 2) V1x) (ap (ap c_{2E}real_{2E}real_{2E}ge V1x) V0a)))))) \quad (25)$$

Assume the following.

$$(\forall V0f \in (ty_{2E}realax_{2E}real^{ty_{2E}realax_{2E}real}).(\forall V1s \in (2^{ty_{2E}realax_{2E}real}).(\forall V2t \in (2^{ty_{2E}realax_{2E}real}).(((p (ap c_{2E}real_{2E}topology_{2E}closed V2t)) \wedge ((p (ap (ap c_{2E}real_{2E}topology_{2E}continuous_{2E}V0f) (ap c_{2E}real_{2E}topology_{2E}closure V1s))) \wedge (\forall V3x \in ty_{2E}realax_{2E}real.((p (ap (ap (c_{2E}bool_{2E}IN ty_{2E}realax_{2E}real) V3x) V1s)) \Rightarrow (p (ap (ap (c_{2E}bool_{2E}IN ty_{2E}realax_{2E}real) (ap V0f V3x)) V2t)))))) \Rightarrow (\forall V4x \in ty_{2E}realax_{2E}real.((p (ap (ap (c_{2E}bool_{2E}IN ty_{2E}realax_{2E}real) V4x) (ap c_{2E}real_{2E}topology_{2E}closure V1s))) \Rightarrow (p (ap (ap (c_{2E}bool_{2E}IN ty_{2E}realax_{2E}real) (ap V0f V4x)) V2t)))))) \quad (26)$$

Theorem 1

$$\begin{aligned} & (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in \\ & \quad (2^{ty_2Erealax_2Ereal}).(\forall V2a \in ty_2Erealax_2Ereal.(\\ ((p (ap (ap c_2Ereal_topology_2Econtinuous_on V0f) (ap c_2Ereal_topology_2Eclosure \\ & \quad V1s))) \wedge (\forall V3x \in ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN \\ & \quad ty_2Erealax_2Ereal) V3x) V1s)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\ & \quad V2a) (ap V0f V3x)))))) \Rightarrow (\forall V4x \in ty_2Erealax_2Ereal.((p (\\ & \quad ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V4x) (ap c_2Ereal_topology_2Eclosure \\ & \quad V1s))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte V2a) (ap V0f V4x)))))))))) \end{aligned}$$