

# thm\_2Ereal\_\_topology\_2ECONTINUOUS\_\_ON\_\_AVOID (TMKVHo6VQLhpjokfxyyHYhaK8Cc1GTgdQV7)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 4** We define  $c\_2Ebool\_2E\_EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{1}$$

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{3}$$

**Definition 7** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2E$

Let  $c\_2Ereal\_topology\_2EDist : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \quad (4)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (5)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax}) \quad (6)$$

**Definition 8** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A$ . **if**  $(\exists x \in A.p (ap\ P\ x))$  **then** (the  $(\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ ).

**Definition 9** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E\_40\ (ty$

Let  $c\_2Erealax\_2Etreallt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreallt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal)}) \quad (7)$$

**Definition 10** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

**Definition 11** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in omega \quad (8)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (9)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (10)$$

**Definition 12** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (11)$$

**Definition 13** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 14** We define  $c\_2Ereal\_topology\_2Econtinuous\_on$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax})$

**Definition 15** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_7E$

**Definition 16** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Let  $ty\_2Ereal\_topology\_2Enet : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ereal\_topology\_2Enet\ A0) \quad (12)$$

Let  $c\_2Ereal\_topology\_2Emk\_net : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c\_2Ereal\_topology\_2Emk\_net\ A.27a \in ((ty\_2Ereal\_topology\_2Enet\ A.27a)^{(2^{A.27a})^{A.27a}}) \quad (13)$$

**Definition 17** We define  $c\_2Ereal\_topology\_2Eat$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Ereal\_topology\_2E$

Let  $c\_2Ereal\_topology\_2Enetord : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c\_2Ereal\_topology\_2Enetord\ A.27a \in (((2^{A.27a})^{A.27a})^{(ty\_2Ereal\_topology\_2Enet\ A.27a)}) \quad (14)$$

**Definition 18** We define  $c\_2Ereal\_topology\_2Ewithin$  to be  $\lambda A.27a : \iota.\lambda V0net \in (ty\_2Ereal\_topology\_2E$

**Definition 19** We define  $c\_2Ereal\_topology\_2Eenetlimit$  to be  $\lambda A.27a : \iota.\lambda V0net \in (ty\_2Ereal\_topology\_2E$

**Definition 20** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.$

**Definition 21** We define  $c\_2Ereal\_topology\_2Etrivial\_limit$  to be  $\lambda A.27a : \iota.\lambda V0net \in (ty\_2Ereal\_topology\_2E$

**Definition 22** We define  $c\_2Ereal\_topology\_2Eeventually$  to be  $\lambda A.27a : \iota.\lambda V0p \in (2^{A.27a}).\lambda V1net \in (ty\_2E$

**Definition 23** We define  $c\_2Ereal\_topology\_2E\_2D\_2D\_3E$  to be  $\lambda A.27a : \iota.\lambda V0f \in (ty\_2Erealax\_2Ereal^A$

**Definition 24** We define  $c\_2Ereal\_topology\_2Econtinuous$  to be  $\lambda A.27a : \iota.\lambda V0f \in (ty\_2Erealax\_2Ereal^A$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (18)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (20)$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1s \in (2^{ty\_2Erealax\_2Ereal}).((p (ap (ap c\_2Ereal\_topology\_2Econtinuous\_on V0f) V1s)) \Leftrightarrow (\forall V2x \in ty\_2Erealax\_2Ereal.((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V2x) V1s)) \Rightarrow (p (ap (ap (c\_2Ereal\_topology\_2Econtinuous ty\_2Erealax\_2Ereal) V0f) (ap (ap (c\_2Ereal\_topology\_2Ewithin ty\_2Erealax\_2Ereal) (ap c\_2Ereal\_topology\_2Eat V2x)) V1s)))))))))) \quad (21)$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1x \in ty\_2Erealax\_2Ereal.(\forall V2s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V3a \in ty\_2Erealax\_2Ereal.(((p (ap (ap (c\_2Ereal\_topology\_2Econtinuous ty\_2Erealax\_2Ereal) V0f) (ap (ap (c\_2Ereal\_topology\_2Ewithin ty\_2Erealax\_2Ereal) (ap c\_2Ereal\_topology\_2Eat V1x)) V2s))) \wedge ((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V1x) V2s)) \wedge (\neg((ap V0f V1x) = V3a)))) \Rightarrow (\exists V4e \in ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V4e)) \wedge (\forall V5y \in ty\_2Erealax\_2Ereal.(((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V5y) V2s)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_topology\_2EDist (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) V1x) V5y))) V4e)))) \Rightarrow (\neg((ap V0f V5y) = V3a)))))))))) \quad (22)$$

**Theorem 1**

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1x \in ty\_2Erealax\_2Ereal.(\forall V2s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V3a \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Ereal\_topology\_2Econtinuous\_on V0f) V2s)) \wedge ((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V1x) V2s)) \wedge (\neg((ap V0f V1x) = V3a)))) \Rightarrow (\exists V4e \in ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V4e)) \wedge (\forall V5y \in ty\_2Erealax\_2Ereal.(((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V5y) V2s)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_topology\_2EDist (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) V1x) V5y))) V4e)))) \Rightarrow (\neg((ap V0f V5y) = V3a)))))))))) \quad (23)$$