

# thm\_2Ereal\_\_topology\_2ECONTINUOUS\_\_ON\_\_CASES\_\_LE (TMTfZZT3WC6UGacfYgy2qrDLf8hNrY1kT59)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2E$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge p))$  of type  $\iota \Rightarrow \iota$ .

**Definition 4** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (1)$$

**Definition 5** We define  $c\_2Ebool\_2E\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 6** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 7** We define  $c\_2Ebool\_2E\_2E21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 8** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty ty\_2Ehreal\_2Ehreal \quad (2)$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty ty\_2Erealax\_2Ereal \quad (3)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)}) ty\_2Erealax\_2Ereal) \quad (4)$$

**Definition 9** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E.40 (ty$   
Let  $c\_2Erealax\_2Etreall\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_lt \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal)) \quad (5)$$

**Definition 10** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 11** We define  $c\_2Ebool\_2E.21$  to be  $(ap (c\_2Ebool\_2E.21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 12** We define  $c\_2Ebool\_2E.7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E.3D\_3D\_3E V0t) c\_2Ebool\_2E.7E))$

**Definition 13** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 14** We define  $c\_2Ereal\_2Ereal\_ge$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 15** We define  $c\_2Ebool\_2E.2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E.21 2) (\lambda V2t \in 2.V2t))))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (6)$$

**Definition 16** We define  $c\_2Epair\_2E.2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epair\_2E.2C x y))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \quad (7)$$

**Definition 17** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E.2E)$ .

**Definition 18** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Epred\_set\_2EDIFF s t))$

Let  $c\_2Ereal\_topology\_2EDist : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal)}) \quad (8)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (9)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty ty\_2Enum\_2Enum \quad (10)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (11)$$

**Definition 19** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (12)$$

**Definition 20** We define  $c\_2Ereal\_topology\_2EOpen$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap\ (c\_2Ebool\_2E2$

**Definition 21** We define  $c\_2Ereal\_topology\_2EClosed$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap\ c\_2Ereal\_topo$

**Definition 22** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

**Definition 23** We define  $c\_2Ereal\_topology\_2Econtinuous\_on$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Ereal$

**Definition 24** We define  $c\_2Ebool\_2E5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V2t \in$

**Definition 25** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (13)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})}) \quad (14)$$

**Definition 26** We define  $c\_2Ereal\_topology\_2Eeuclidean$  to be  $(ap\ (c\_2Etopology\_2Etopology\ ty\_2Erealax$

Let  $c\_2Etopology\_2Eopen\_in : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Eopen\_in\ A\_27a \in ((2^{(2^{A\_27a})})^{(ty\_2Etopology\_2Etopology\ A\_27a)}) \quad (15)$$

**Definition 27** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c$

**Definition 28** We define  $c\_2Ereal\_topology\_2Esubtopology$  to be  $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopo$

**Definition 29** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_s$

**Definition 30** We define  $c\_2Etopology\_2Etopspace$  to be  $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology$

**Definition 31** We define  $c\_2Etopology\_2Eclosed\_in$  to be  $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (18)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (19)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge (p V1t2) \wedge (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3))))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2. (\neg((p V0t) \wedge (\neg(p V0t))))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (25)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (26)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a. (V0x = V0x)) \quad (27)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (29)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1Q \in 2. ((\exists V2x \in A\_27a. ((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ V1Q))) \Leftrightarrow ((\forall V3x \in A\_27a. (p\ (ap\ V0P\ V3x)) \Rightarrow (p\ V1Q)))))) \quad (30)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A\_27a}). ((\exists V2x \in A\_27a. ((p\ V0P) \Rightarrow (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \Rightarrow (\exists V3x \in A\_27a. (p\ (ap\ V1Q\ V3x)))))) \quad (31)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (32)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \quad (33)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1a \in A\_27a. ((\exists V2x \in A\_27a. ((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a)))) \quad (34)$$

Assume the following.

$$(\forall V0r \in 2. (\forall V1p \in 2. (\forall V2q \in 2. (((p\ V1p) \wedge (p\ V2q) \Rightarrow (p\ V0r)) \Leftrightarrow ((p\ V1p) \Rightarrow ((p\ V2q) \Rightarrow (p\ V0r)))))) \quad (35)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27b. (\forall V2a \in A\_27a. (\forall V3b \in A\_27b. (((ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \quad (36)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\ & (2^{A\_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a. ((p (ap (ap (c\_2Ebool\_2EIN \\ & A\_27a) V2x) V0s)) \Leftrightarrow (p (ap (ap (c\_2Ebool\_2EIN\ A\_27a) V2x) V1t))))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}). (\forall V1v \in \\ & A\_27a. ((p (ap (ap (c\_2Ebool\_2EIN\ A\_27a) V1v) (ap (c\_2Epred\_set\_2EGSPEC \\ & A\_27a\ A\_27b) V0f))) \Leftrightarrow (\exists V2x \in A\_27b. ((ap (ap (c\_2Epair\_2E\_2C \\ & A\_27a\ 2) V1v) c\_2Ebool\_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\ & (2^{A\_27a}). (\forall V2x \in A\_27a. ((p (ap (ap (c\_2Ebool\_2EIN\ A\_27a) \\ & V2x) (ap (ap (c\_2Epred\_set\_2EUNION\ A\_27a) V0s) V1t))) \Leftrightarrow ((p (ap \quad (39) \\ & (ap (c\_2Ebool\_2EIN\ A\_27a) V2x) V0s)) \vee (p (ap (ap (c\_2Ebool\_2EIN \\ & A\_27a) V2x) V1t)))))) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealx\_2Ereal. (\forall V1y \in ty\_2Erealx\_2Ereal. \\ & ((p (ap (ap\ c\_2Ereal\_2Ereal\_lte\ V0x) V1y)) \vee (p (ap (ap\ c\_2Ereal\_2Ereal\_lte \\ & V1y) V0x)))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealx\_2Ereal. (\forall V1y \in ty\_2Erealx\_2Ereal. \\ & (((p (ap (ap\ c\_2Ereal\_2Ereal\_lte\ V0x) V1y)) \wedge (p (ap (ap\ c\_2Ereal\_2Ereal\_lte \\ & V1y) V0x))) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & (\forall V0f \in (ty\_2Erealx\_2Ereal^{ty\_2Erealx\_2Ereal}). (\forall V1s \in \\ & (2^{ty\_2Erealx\_2Ereal}). (\forall V2t \in (2^{ty\_2Erealx\_2Ereal}). \\ & (((p (ap (ap\ c\_2Ereal\_topology\_2Econtinuous\_on\ V0f) V1s)) \wedge \\ & (p (ap (ap (c\_2Epred\_set\_2ESUBSET\ ty\_2Erealx\_2Ereal) V2t) V1s))) \Rightarrow \\ & (p (ap (ap\ c\_2Ereal\_topology\_2Econtinuous\_on\ V0f) V2t)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1s \in \\
& \quad (2^{ty\_2Erealax\_2Ereal}).(\forall V2t \in (2^{ty\_2Erealax\_2Ereal}). \\
& \quad ((p (ap (ap c\_2Ereal\_topology\_2Econtinuous\_on V0f) V1s)) \wedge \\
& (p (ap c\_2Ereal\_topology\_2EClosed V2t))) \Rightarrow (p (ap (ap (c\_2Etopology\_2EClosed\_in \\
& \quad ty\_2Erealax\_2Ereal) (ap (ap (c\_2Ereal\_topology\_2Esubtopology \\
& \quad ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Euclidean) V1s)) \\
& \quad (ap (c\_2Epred\_set\_2EGSPEC ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& \quad (\lambda V3x \in ty\_2Erealax\_2Ereal.(ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& 2) V3x) (ap (ap c\_2Ebool\_2E\_2F\_5C (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\
& \quad V3x) V1s)) (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) (ap V0f V3x)) \\
& \quad V2t))))))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty\_2Erealax\_2Ereal.(p (ap c\_2Ereal\_topology\_2EClosed \\
& (ap (c\_2Epred\_set\_2EGSPEC ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& (\lambda V1x \in ty\_2Erealax\_2Ereal.(ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& \quad 2) V1x) (ap (ap c\_2Ereal\_2Ereal\_lte V1x) V0a))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty\_2Erealax\_2Ereal.(p (ap c\_2Ereal\_topology\_2EClosed \\
& (ap (c\_2Epred\_set\_2EGSPEC ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& (\lambda V1x \in ty\_2Erealax\_2Ereal.(ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& \quad 2) V1x) (ap (ap c\_2Ereal\_2Ereal\_ge V1x) V0a))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty\_2Erealax\_2Ereal}). (\forall V1f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). \\
& \quad (\forall V2g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). (\forall V3s \in \\
& \quad \quad (2^{ty\_2Erealax\_2Ereal}). (\forall V4t \in (2^{ty\_2Erealax\_2Ereal}). \\
& \quad \quad \quad (((p (ap (ap (c\_2Etopology\_2Eclosed\_in ty\_2Erealax\_2Ereal) \\
& \quad \quad \quad (ap (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\
& \quad \quad \quad c\_2Ereal\_topology\_2Eeuclidean) (ap (ap (c\_2Epred\_set\_2EUNION \\
& \quad \quad \quad ty\_2Erealax\_2Ereal) V3s) V4t))) V3s)) \wedge ((p (ap (ap (c\_2Etopology\_2Eclosed\_in \\
& \quad \quad \quad ty\_2Erealax\_2Ereal) (ap (ap (c\_2Ereal\_topology\_2Esubtopology \\
& \quad \quad \quad ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Eeuclidean) (ap (ap \\
& \quad \quad \quad (c\_2Epred\_set\_2EUNION ty\_2Erealax\_2Ereal) V3s) V4t))) V4t)) \wedge \\
& \quad \quad \quad ((p (ap (ap c\_2Ereal\_topology\_2Econtinuous\_on V1f) V3s)) \wedge \\
& \quad \quad \quad (p (ap (ap c\_2Ereal\_topology\_2Econtinuous\_on V2g) V4t)) \wedge (\forall V5x \in \\
& \quad \quad \quad ty\_2Erealax\_2Ereal. (((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\
& \quad \quad \quad V5x) V3s)) \wedge (\neg(p (ap V0P V5x)))) \vee ((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\
& \quad \quad \quad V5x) V4t)) \wedge (p (ap V0P V5x)))))) \Rightarrow ((ap V1f V5x) = (ap V2g V5x)))))) \Rightarrow \\
& \quad \quad \quad (p (ap (ap c\_2Ereal\_topology\_2Econtinuous\_on (\lambda V6x \in ty\_2Erealax\_2Ereal. \\
& \quad \quad \quad (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Erealax\_2Ereal) (ap V0P V6x)) \\
& \quad \quad \quad (ap V1f V6x)) (ap V2g V6x)))) (ap (ap (c\_2Epred\_set\_2EUNION ty\_2Erealax\_2Ereal) \\
& \quad \quad \quad V3s) V4t)))))))))
\end{aligned} \tag{46}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{47}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{50}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& \quad (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& \quad p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& \quad ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{52}$$



Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow ( \\
& (p \vee V1q) \wedge (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee (\neg(p \vee V1q)) \vee (\neg(p \vee V2r)))) \wedge (((p \vee V1q) \vee \\
& (\neg(p \vee V0p))) \wedge ((p \vee V2r) \vee (\neg(p \vee V0p))))))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow ( \\
& (p \vee V1q) \vee (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee (\neg(p \vee V1q))) \wedge ((p \vee V0p) \vee (\neg(p \vee V2r)))) \wedge \\
& ((p \vee V1q) \vee ((p \vee V2r) \vee (\neg(p \vee V0p))))))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow ( \\
& (p \vee V1q) \Rightarrow (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee (p \vee V1q)) \wedge (((p \vee V0p) \vee (\neg(p \vee V2r))) \wedge ( \\
& \neg(p \vee V1q)) \vee ((p \vee V2r) \vee (\neg(p \vee V0p))))))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \vee V0p) \Leftrightarrow (\neg(p \vee V1q))) \Leftrightarrow (((p \vee V0p) \vee \\
& (p \vee V1q)) \wedge ((\neg(p \vee V1q)) \vee (\neg(p \vee V0p))))))
\end{aligned} \tag{56}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \Rightarrow (p \vee V1q))) \Rightarrow (p \vee V0p))) \tag{57}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \Rightarrow (p \vee V1q))) \Rightarrow (\neg(p \vee V1q)))) \tag{58}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \vee (p \vee V1q))) \Rightarrow (\neg(p \vee V0p)))) \tag{59}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \vee (p \vee V1q))) \Rightarrow (\neg(p \vee V1q)))) \tag{60}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \vee V0p))) \Rightarrow (p \vee V0p))) \tag{61}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in \\
& (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2h \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). \\
& (\forall V3s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V4a \in ty\_2Erealax\_2Ereal. \\
& (((p (ap (ap (ap c\_2Ereal\_topology\_2Econtinuous\_on V0f) (ap (c\_2Epred\_set\_2EGSPEC \\
& ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) (\lambda V5t \in ty\_2Erealax\_2Ereal. \\
& (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal 2) V5t) (ap (ap c\_2Ebool\_2E\_2F\_5C \\
& (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V5t) V3s)) (ap (ap c\_2Ereal\_2Ereal\_lte \\
& (ap V2h V5t)) V4a))))))\wedge((p (ap (ap c\_2Ereal\_topology\_2Econtinuous\_on \\
& V1g) (ap (c\_2Epred\_set\_2EGSPEC ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& (\lambda V6t \in ty\_2Erealax\_2Ereal.(ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& 2) V6t) (ap (ap c\_2Ebool\_2E\_2F\_5C (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\
& V6t) V3s)) (ap (ap c\_2Ereal\_2Ereal\_lte V4a) (ap V2h V6t))))))\wedge \\
& ((p (ap (ap c\_2Ereal\_topology\_2Econtinuous\_on V2h) V3s))\wedge( \\
& \forall V7t \in ty\_2Erealax\_2Ereal.(((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\
& V7t) V3s))\wedge((ap V2h V7t) = V4a))\Rightarrow((ap V0f V7t) = (ap V1g V7t))))))\Rightarrow \\
& (p (ap (ap c\_2Ereal\_topology\_2Econtinuous\_on (\lambda V8t \in ty\_2Erealax\_2Ereal. \\
& (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Erealax\_2Ereal) (ap (ap c\_2Ereal\_2Ereal\_lte \\
& (ap V2h V8t)) V4a)) (ap V0f V8t)) (ap V1g V8t)))) V3s))))))
\end{aligned}$$