

thm_2Ereal__topology_2ECONTINUOUS__ON__COMPONENTS__I (TMQJqfAjqZf9h7LFknAe5heXUfaydUTKT78)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 3 We define $c_2Ecombin_2EC$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_21$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a})).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1P \in (2^{A_27a}))))$

Definition 6 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in (A_27b^{A_27c}).\lambda V1g \in (A_27c^{A_27a}))$

Definition 7 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a})).(ap V1f V0x)))$

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (2)$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS_prod A_27a A_27b) (ty_2Epair_2Eprod A_27a A_27b)))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (3)$$

Definition 11 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$
Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \quad (4)$$

Definition 12 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 13 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 14 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E7E))$

Definition 15 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2E$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealx_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealx_2Ereal\ ty_2Erealx_2Ereal)}) \quad (5)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (6)$$

Let $c_2Erealx_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealx_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealx_2Ereal}) \quad (7)$$

Definition 16 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p\ x))$
of type $\iota \Rightarrow \iota$.

Definition 17 We define $c_2Erealx_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealx_2Ereal.(ap\ (c_2Emin_2E40\ (ap\ P\ x)))$

Let $c_2Erealx_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealx_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (8)$$

Definition 18 We define $c_2Erealx_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal.\lambda V1T2 \in ty_2Erealx_2Ereal.$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \quad (9)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (11)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A_27a}).((\forall V2x \in A_27a.((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A_27a.(p (ap V1P V3x))) \vee (p V0Q))))) \quad (30)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))) \quad (31)$$

Assume the following.

$$2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow 2.(((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1a \in A_27a.((\exists V2x \in A_27a.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27b.(\forall V2a \in A_27a.(\forall V3b \in A_27b.(((ap (ap (c_2Epair_2E_2C A_27a A_27b) V0x) V1y) = (ap (ap (c_2Epair_2E_2C A_27a A_27b) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \quad (34)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}). (\forall V1v \in \\ & A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\ & A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap\ (ap\ (c_2Epair_2E_2C \\ & A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0P \in (2^{A_27a}). (\forall V1f \in (A_27a^{A_27b}). (\forall V2s \in \\ & (2^{A_27b}). ((\forall V3y \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ \\ & V3y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_27b\ A_27a)\ V1f)\ V2s))) \Rightarrow (\\ & p\ (ap\ V0P\ V3y)))) \Leftrightarrow (\forall V4x \in A_27b. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27b)\ V4x)\ V2s)) \Rightarrow (p\ (ap\ V0P\ (ap\ V1f\ V4x)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0s \in (2^{A_27a}). ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ \\ & V0s)) \Rightarrow (\forall V1f \in (A_27b^{A_27a}). (p\ (ap\ (c_2Epred_set_2EFINITE \\ & A_27b)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_27a\ A_27b)\ V1f)\ V0s)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1sos \in \\ & (2^{(2^{A_27a})}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (ap\ (c_2Epred_set_2EBIGUNION \\ & A_27a)\ V1sos))) \Leftrightarrow (\exists V2s \in (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V0x)\ V2s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a})\ V2s)\ V1sos)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & (\forall V0s \in (2^{ty_2Erealx_2Ereal}). (\forall V1t \in (2^{ty_2Erealx_2Ereal}). \\ & (\forall V2u \in (2^{ty_2Erealx_2Ereal}). (((p\ (ap\ (ap\ (c_2Etopology_2Eclosed_in \\ & ty_2Erealx_2Ereal)\ (ap\ (ap\ (c_2Ereal_topology_2Esubtopology \\ & ty_2Erealx_2Ereal)\ c_2Ereal_topology_2Euclidean)\ V1t)) \\ & V0s)) \wedge (p\ (ap\ (ap\ (c_2Etopology_2Eclosed_in\ ty_2Erealx_2Ereal) \\ & (ap\ (ap\ (c_2Ereal_topology_2Esubtopology\ ty_2Erealx_2Ereal) \\ & c_2Ereal_topology_2Euclidean)\ V2u))\ V1t)))) \Rightarrow (p\ (ap\ (ap\ (c_2Etopology_2Eclosed_in \\ & ty_2Erealx_2Ereal)\ (ap\ (ap\ (c_2Ereal_topology_2Esubtopology \\ & ty_2Erealx_2Ereal)\ c_2Ereal_topology_2Euclidean)\ V2u)) \\ & V0s)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in \\
& (2^{ty_2Erealax_2Ereal}).((p (ap (ap c_2Ereal_topology_2Econtinuous_on \\
V0f) V1s)) \Leftrightarrow (\forall V2t \in (2^{ty_2Erealax_2Ereal}).((p (ap c_2Ereal_topology_2EClosed \\
V2t)) \Rightarrow (p (ap (ap (c_2Etopology_2Eclosed_in ty_2Erealax_2Ereal) \\
(ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
c_2Ereal_topology_2Eeuclidean) V1s)) (ap (c_2Epred_set_2EGSPEC \\
ty_2Erealax_2Ereal ty_2Erealax_2Ereal) (\lambda V3x \in ty_2Erealax_2Ereal. \\
(ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal 2) V3x) (ap (ap c_2Ebool_2E_2F_5C \\
(ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V3x) V1s)) (ap (ap (\\
c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap V0f V3x)) V2t))))))))))))) \\
& \hspace{15em} (41)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow \forall A.27e.nonempty \\
& A.27e \Rightarrow \forall A.27f.nonempty\ A.27f \Rightarrow \forall A.27g.nonempty\ A.27g \Rightarrow \\
& \forall A.27h.nonempty\ A.27h \Rightarrow \forall A.27i.nonempty\ A.27i \Rightarrow (\\
& (\forall V0P \in (2^{A.27a}).(\forall V1f \in ((2^{A.27b})^{A.27a}).((ap \\
& (c.2Epred_set_2EBIGUNION\ A.27b)\ (ap\ (c.2Epred_set_2EGSPEC \\
& (2^{A.27b})\ A.27a)\ (\lambda V2x \in A.27a.(ap\ (ap\ (c.2Epair_2E_2C\ (2^{A.27b}) \\
& 2)\ (ap\ V1f\ V2x))\ (ap\ V0P\ V2x)))))) = (ap\ (c.2Epred_set_2EGSPEC\ A.27b \\
& A.27b)\ (\lambda V3a \in A.27b.(ap\ (ap\ (c.2Epair_2E_2C\ A.27b\ 2)\ V3a)\ (\\
& ap\ (c.2Ebool_2E_3F\ A.27a)\ (\lambda V4x \in A.27a.(ap\ (ap\ c.2Ebool_2E_2F_5C \\
& (ap\ V0P\ V4x))\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27b)\ V3a)\ (ap\ V1f\ V4x)))))))))) \wedge \\
& ((\forall V5P \in ((2^{A.27d})^{A.27c}).(\forall V6f \in (((2^{A.27e})^{A.27d})^{A.27c}). \\
& ((ap\ (c.2Epred_set_2EBIGUNION\ A.27e)\ (ap\ (c.2Epred_set_2EGSPEC \\
& (2^{A.27e})\ ty_2Epair_2Eprod\ A.27c\ A.27d))\ (ap\ (c.2Epair_2EUNCURRY \\
& A.27c\ A.27d\ (ty_2Epair_2Eprod\ (2^{A.27e})\ 2))\ (\lambda V7x \in A.27c. \\
& (\lambda V8y \in A.27d.(ap\ (ap\ (c.2Epair_2E_2C\ (2^{A.27e})\ 2)\ (ap\ (ap\ V6f \\
& V7x)\ V8y))\ (ap\ (ap\ V5P\ V7x)\ V8y)))))) = (ap\ (c.2Epred_set_2EGSPEC \\
& A.27e\ A.27e)\ (\lambda V9a \in A.27e.(ap\ (ap\ (c.2Epair_2E_2C\ A.27e\ 2)\ \\
& V9a)\ (ap\ (c.2Ebool_2E_3F\ A.27c)\ (\lambda V10x \in A.27c.(ap\ (c.2Ebool_2E_3F \\
& A.27d)\ (\lambda V11y \in A.27d.(ap\ (ap\ c.2Ebool_2E_2F_5C\ (ap\ (ap\ V5P\ V10x) \\
& V11y))\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27e)\ V9a)\ (ap\ (ap\ V6f\ V10x)\ V11y)))))))))) \wedge \\
& (\forall V12P \in (((2^{A.27h})^{A.27g})^{A.27f}).(\forall V13f \in (((2^{A.27i})^{A.27h})^{A.27g})^{A.27f}). \\
& ((ap\ (c.2Epred_set_2EBIGUNION\ A.27i)\ (ap\ (c.2Epred_set_2EGSPEC \\
& (2^{A.27i})\ ty_2Epair_2Eprod\ A.27f\ (ty_2Epair_2Eprod\ A.27g\ A.27h))) \\
& (ap\ (c.2Epair_2EUNCURRY\ A.27f\ (ty_2Epair_2Eprod\ A.27g\ A.27h) \\
& (ty_2Epair_2Eprod\ (2^{A.27i})\ 2))\ (\lambda V14x \in A.27f.(ap\ (c.2Epair_2EUNCURRY \\
& A.27g\ A.27h\ (ty_2Epair_2Eprod\ (2^{A.27i})\ 2))\ (\lambda V15y \in A.27g. \\
& (\lambda V16z \in A.27h.(ap\ (ap\ (c.2Epair_2E_2C\ (2^{A.27i})\ 2)\ (ap\ (ap \\
& (ap\ V13f\ V14x)\ V15y)\ V16z))\ (ap\ (ap\ (ap\ V12P\ V14x)\ V15y)\ V16z)))))) = \\
& (ap\ (c.2Epred_set_2EGSPEC\ A.27i\ A.27i)\ (\lambda V17a \in A.27i.(ap \\
& (ap\ (c.2Epair_2E_2C\ A.27i\ 2)\ V17a)\ (ap\ (c.2Ebool_2E_3F\ A.27f) \\
& (\lambda V18x \in A.27f.(ap\ (c.2Ebool_2E_3F\ A.27g)\ (\lambda V19y \in A.27g. \\
& (ap\ (c.2Ebool_2E_3F\ A.27h)\ (\lambda V20z \in A.27h.(ap\ (ap\ c.2Ebool_2E_2F_5C \\
& (ap\ (ap\ (ap\ V12P\ V18x)\ V19y)\ V20z))\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27i) \\
& V17a)\ (ap\ (ap\ (ap\ V13f\ V18x)\ V19y)\ V20z)))))))))))))
\end{aligned} \tag{42}$$

Assume the following.

$$(\forall V0u \in (2^{ty_2Erealax_2Ereal}).(V0u = (ap\ (c.2Epred_set_2EBIGUNION\ ty_2Erealax_2Ereal)\ (ap\ c.2Ereal_topology_2Ecomponents\ V0u)))) \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}). (\forall V1c \in (2^{ty_2Erealax_2Ereal}). \\
& ((p (ap (ap (c_2Ebool_2EIN (2^{ty_2Erealax_2Ereal}) V1c) (ap c_2Ereal_topology_2Ecomponents \\
& V0s))) \Rightarrow (p (ap (ap (c_2Etopology_2Eclosed_in ty_2Erealax_2Ereal) \\
& (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& c_2Ereal_topology_2Eeuclidean) V0s)) V1c))))))
\end{aligned} \tag{44}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{45}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{48}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& p V2r) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{52}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\neg(p V1q) \vee ((p V2r) \vee \neg(p V0p)))))))) \quad (53)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (\neg(p V1q) \vee \neg(p V0p)))))) \quad (54)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow \neg(p V1q))) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V0p))) \quad (57)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V1q))) \quad (58)$$

Assume the following.

$$(\forall V0p \in 2. (\neg(\neg(p V0p))) \Rightarrow (p V0p)) \quad (59)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0top \in (ty_2Etopology_2Etopology A.27a). (\forall V1s \in (2^{(2^{A-27a})}). (((p (ap (c_2Epred_set_2EFINITE (2^{A-27a}) V1s)) \wedge (\forall V2t \in (2^{A-27a}). ((p (ap (ap (c_2Ebool_2EIN (2^{A-27a}) V2t) V1s)) \Rightarrow (p (ap (ap (c_2Etopology_2Eclosed_in A.27a) V0top) V2t)))))) \Rightarrow (p (ap (ap (c_2Etopology_2Eclosed_in A.27a) V0top) (ap (c_2Epred_set_2EBIGUNION A.27a) V1s)))))) \quad (60)$$

Theorem 1

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V1s \in (2^{ty_2Erealax_2Ereal}). (((p (ap (c_2Epred_set_2EFINITE (2^{ty_2Erealax_2Ereal}) (ap c_2Ereal_topology_2Ecomponents V1s))) \wedge (\forall V2c \in (2^{ty_2Erealax_2Ereal}). ((p (ap (ap (c_2Ebool_2EIN (2^{ty_2Erealax_2Ereal}) V2c) (ap c_2Ereal_topology_2Ecomponents V1s))) \Rightarrow (p (ap (ap c_2Ereal_topology_2Econtinuous_on V0f) V2c)))))) \Rightarrow (p (ap (ap c_2Ereal_topology_2Econtinuous_on V0f) V1s))))))$$