

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}})$$
 (3)

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal$$
 (4)

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealx_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealx_2Ereal\ ty_2Erealx_2Ereal)})$$
 (5)

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal$$
 (6)

Let $c_2Erealx_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealx_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealx_2Ereal})$$
 (7)

Definition 10 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p\ (ap\ P\ x))$ then $(the\ (\lambda x.x \in A \wedge P\ x))$ of type $\iota \Rightarrow \iota$.

Definition 11 We define $c_2Erealx_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealx_2Ereal.(ap\ (c_2Emin_2E40\ (the\ (\lambda x.x \in A \wedge P\ x))))$

Let $c_2Erealx_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealx_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)})$$
 (8)

Definition 12 We define $c_2Erealx_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal.\lambda V1T2 \in ty_2Erealx_2Ereal.(c_2Erealx_2Etreall_lt\ T1\ T2)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega$$
 (9)

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum$$
 (10)

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega})$$
 (11)

Definition 13 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum})$$
 (12)

Definition 14 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 15 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}). (ap\ (c_2Ebool_2E_2$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (13)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^A-27a)})}) \quad (14)$$

Definition 16 We define $c_2Ereal_topology_2Eeuclidean$ to be $(ap\ (c_2Etopology_2Etopology\ ty_2Erealax_2Ereal$

Let $c_2Etopology_2Eopen_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Eopen_in\ A_27a \in ((2^{(2^A-27a)})^{(ty_2Etopology_2Etopology\ A_27a)}) \quad (15)$$

Definition 17 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap\ (c_2E$

Definition 18 We define $c_2Ereal_topology_2Esubtopology$ to be $\lambda A_27a : \iota. \lambda V0top \in (ty_2Etopology_2Etopology$

Definition 19 We define $c_2Ereal_topology_2Econtinuous_on$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal$

Definition 20 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A-27a}). \lambda V1s \in (A_27a^{A-27a}).$

Definition 21 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap\ (c_2E$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A-27a}). (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a)\ V0s)\ V0s))) \quad (18)$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in \\
& \quad (2^{ty_2Erealax_2Ereal}).(\forall V2t \in (2^{ty_2Erealax_2Ereal}). \\
& \quad ((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) (ap (\\
& \quad ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& \quad V0f) V1s)) V2t)) \Rightarrow ((p (ap (ap c_2Ereal_topology_2Econtinuous_on \\
& \quad V0f) V1s)) \Leftrightarrow (\forall V3u \in (2^{ty_2Erealax_2Ereal}).((p (ap (ap (\\
& \quad c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& \quad ty_2Erealax_2Ereal) c_2Ereal_topology_2Eeuclidean) V2t)) \\
& \quad V3u)) \Rightarrow (p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) \\
& \quad (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& \quad c_2Ereal_topology_2Eeuclidean) V1s)) (ap (c_2Epred_set_2EGSPEC \\
& \quad ty_2Erealax_2Ereal ty_2Erealax_2Ereal) (\lambda V4x \in ty_2Erealax_2Ereal. \\
& \quad (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal 2) V4x) (ap (ap c_2Ebool_2E_2F_5C \\
& \quad (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V4x) V1s)) (ap (ap (\\
& \quad c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap V0f V4x)) V3u))))))))))))) \\
& \hspace{15em} (19)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in \\
& \quad (2^{ty_2Erealax_2Ereal}).((p (ap (ap c_2Ereal_topology_2Econtinuous_on \\
& \quad V0f) V1s)) \Leftrightarrow (\forall V2t \in (2^{ty_2Erealax_2Ereal}).((p (ap (ap (\\
& \quad c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& \quad ty_2Erealax_2Ereal) c_2Ereal_topology_2Eeuclidean) (ap (ap \\
& \quad (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& \quad V0f) V1s))) V2t)) \Rightarrow (p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) \\
& \quad (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& \quad c_2Ereal_topology_2Eeuclidean) V1s)) (ap (c_2Epred_set_2EGSPEC \\
& \quad ty_2Erealax_2Ereal ty_2Erealax_2Ereal) (\lambda V3x \in ty_2Erealax_2Ereal. \\
& \quad (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal 2) V3x) (ap (ap c_2Ebool_2E_2F_5C \\
& \quad (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V3x) V1s)) (ap (ap (\\
& \quad c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap V0f V3x)) V2t)))))))))))))
\end{aligned}$$