

# thm\_2Ereal\_\_topology\_2ECONTINUOUS\_\_ON\_\_UNION\_\_LOCAL (TMNTZuMbsA2g6c14nPvzc53JLn51zfJUBNS)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ecombin\_2E\_2K$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 4** We define  $c\_2Ecombin\_2E\_2S$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 5** We define  $c\_2Ecombin\_2E\_2I$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2E\_2S A\_27a (A\_27a^{A\_27a})) A\_27a))$

**Definition 6** We define  $c\_2Ebool\_2E\_2IN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 7** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) P)))$

**Definition 9** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Emin\_2E\_3D (2^{A\_27a})) (V1t \subseteq V0s))$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{1}$$

**Definition 10** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (V1t2 \subseteq V0t1))))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{3}$$

**Definition 11** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c\_2Ereal\_topology\_2EDist : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \quad (4)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (5)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax}) \quad (6)$$

**Definition 12** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ **if**  $(\exists x \in A.p (ap\ P\ x))$  **then**  $(the (\lambda x.x \in A \wedge p\ x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 13** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E\_40 (the (\lambda x.x \in A \wedge p\ x))))$

Let  $c\_2Erealax\_2Etrealm\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (7)$$

**Definition 14** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E\_40 (the (\lambda x.x \in A \wedge p\ x))))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in omega \quad (8)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (9)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (10)$$

**Definition 15** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (11)$$

**Definition 16** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A.27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40 (the (\lambda x.x \in A \wedge p\ x))))))$

**Definition 17** We define  $c\_2Ereal\_topology\_2EOpen$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap\ (c\_2Ebool\_2E\_3F (the (\lambda x.x \in A \wedge p\ x))))$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (12)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in \\ ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^A-27a)})}) \quad (13)$$

**Definition 18** We define  $c\_2Ereal\_topology\_2Euclidean$  to be  $(ap\ (c\_2Etopology\_2Etopology\ ty\_2Erealax$

Let  $c\_2Etopology\_2Eopen\_in : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Eopen\_in\ A\_27a \in \\ ((2^{(2^A-27a)})^{(ty\_2Etopology\_2Etopology\ A\_27a)}) \quad (14)$$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a\ A\_27b \in ((2^{A-27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A-27b}}) \quad (15)$$

**Definition 19** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ (c\_2$

**Definition 20** We define  $c\_2Ereal\_topology\_2Esubtopology$  to be  $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology$

**Definition 21** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^A-27a)}).(ap\ (c\_2Epred\_set$

**Definition 22** We define  $c\_2Etopology\_2Etopspace$  to be  $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology$

**Definition 23** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 24** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E$

**Definition 25** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ (c\_2$

**Definition 26** We define  $c\_2Etopology\_2Eclosed\_in$  to be  $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology$

**Definition 27** We define  $c\_2Ereal\_topology\_2Elimit\_point\_of$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1s \in ($

Let  $ty\_2Ereal\_topology\_2Enet : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ereal\_topology\_2Enet\ A0) \quad (16)$$

Let  $c\_2Ereal\_topology\_2Enetord : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ereal\_topology\_2Enetord\ A\_27a \in \\ (((2^{A-27a})^{A-27a})^{(ty\_2Ereal\_topology\_2Enet\ A\_27a)}) \quad (17)$$

**Definition 28** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 29** We define  $c\_2Ereal\_topology\_2Etrivial\_limit$  to be  $\lambda A.27a : \iota.\lambda V0net \in (ty\_2Ereal\_topology\_2Etrivial\_limit)$

**Definition 30** We define  $c\_2Ereal\_topology\_2Econtinuous\_on$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})$

**Definition 31** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A.27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c\_2Epred\_set\_2EUNION))$

**Definition 32** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $c\_2Ereal\_topology\_2Emk\_net : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow c\_2Ereal\_topology\_2Emk\_net \\ & A.27a \in ((ty\_2Ereal\_topology\_2Enet\ A.27a)^{(2^{A-27a})^{A-27a}}) \end{aligned} \quad (18)$$

**Definition 33** We define  $c\_2Ereal\_topology\_2Eat$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Ereal\_topology\_2Eat))$

**Definition 34** We define  $c\_2Ereal\_topology\_2Ewithin$  to be  $\lambda A.27a : \iota.\lambda V0net \in (ty\_2Ereal\_topology\_2Ewithin)$

**Definition 35** We define  $c\_2Ereal\_topology\_2Eeventually$  to be  $\lambda A.27a : \iota.\lambda V0p \in (2^{A-27a}).\lambda V1net \in (ty\_2Erealax\_2Ereal)$

**Definition 36** We define  $c\_2Ereal\_topology\_2E\_2D\_2D\_3E$  to be  $\lambda A.27a : \iota.\lambda V0f \in (ty\_2Erealax\_2Ereal^{A-27a})$

Assume the following.

$$True \quad (19)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (20)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A.27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True) \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (26)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (27)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p \ V0t))))) \quad (29)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{A-27a}).((\neg(\forall V1x \in A\_27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\exists V2x \in A\_27a.(\neg(p \ (ap \ V0P \ V2x))))) \quad (30)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{A-27a}).((\neg(\exists V1x \in A\_27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg(p \ (ap \ V0P \ V2x))))) \quad (31)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{A-27a}).(\forall V1Q \in (2^{A-27a}).((\forall V2x \in A\_27a.((p \ (ap \ V0P \ V2x)) \wedge (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((\forall V3x \in A\_27a.(p \ (ap \ V0P \ V3x))) \wedge (\forall V4x \in A\_27a.(p \ (ap \ V1Q \ V4x))))) \quad (32)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A-27a}).(((p \ V0P) \wedge (\forall V2x \in A\_27a.(p \ (ap \ V1Q \ V2x)))) \Leftrightarrow (\forall V3x \in A\_27a.((p \ V0P) \wedge (p \ (ap \ V1Q \ V3x))))) \quad (33)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A-27a}).(((p \ V0P) \vee (\exists V2x \in A\_27a.(p \ (ap \ V1Q \ V2x)))) \Leftrightarrow (\exists V3x \in A\_27a.((p \ V0P) \vee (p \ (ap \ V1Q \ V3x))))) \quad (34)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ 2.((\exists V2x \in A.27a.((p \ (ap \ V0P \ V2x)) \wedge (p \ V1Q)))) \Leftrightarrow ((\exists V3x \in \\ A.27a.(p \ (ap \ V0P \ V3x)) \wedge (p \ V1Q)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ( \\ 2^{A.27a}).((\exists V2x \in A.27a.((p \ V0P) \wedge (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((p \\ V0P) \wedge (\exists V3x \in A.27a.(p \ (ap \ V1Q \ V3x)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in ( \\ 2^{A.27a}).((\forall V2x \in A.27a.((p \ (ap \ V1P \ V2x)) \vee (p \ V0Q))) \Leftrightarrow ((\forall V3x \in \\ A.27a.(p \ (ap \ V1P \ V3x)) \vee (p \ V0Q)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ( \\ 2^{A.27a}).((\forall V2x \in A.27a.((p \ V0P) \vee (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((p \\ V0P) \vee (\forall V3x \in A.27a.(p \ (ap \ V1Q \ V3x)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee ( \\ (p \ V1B) \vee (p \ V2C))) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((p \ V0A) \vee (p \ V1B)) \Leftrightarrow ((p \ V1B) \vee \\ (p \ V0A)))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg( \\ p \ V0A)) \vee (\neg(p \ V1B)))))) \wedge (((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \wedge (\neg(p \ V1B)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow ( \\ \forall V0P \in ((2^{A.27b})^{A.27a}).(\forall V1x \in A.27a.(\exists V2y \in \\ A.27b.(p \ (ap \ (ap \ V0P \ V1x) \ V2y)))) \Leftrightarrow (\exists V3f \in (A.27b^{A.27a}).( \\ \forall V4x \in A.27a.(p \ (ap \ (ap \ V0P \ V4x) \ (ap \ V3f \ V4x)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.((ap \ (c.2Ecombin.2EI \\ A.27a) \ V0x) = V0x)) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in \\ & (2^{A.27a}). (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a) \\ & V2x)\ (ap\ (ap\ (c.2Epred\_set.2EUNION\ A.27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (44) \\ & (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V0s)) \vee (p\ (ap\ (ap\ (c.2Ebool.2EIN \\ & A.27a)\ V2x)\ V1t)))))) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0s \in (2^{ty.2Erealx.2Ereal}). (\forall V1t \in (2^{ty.2Erealx.2Ereal}). \\ & ((p\ (ap\ (ap\ (c.2Etopology.2Eclosed\_in\ ty.2Erealx.2Ereal)\ ( \\ & ap\ (ap\ (c.2Ereal\_topology.2Esubtopology\ ty.2Erealx.2Ereal) \\ & c.2Ereal\_topology.2Eeuclidean)\ V1t))\ V0s)) \Leftrightarrow ((p\ (ap\ (ap\ (c.2Epred\_set.2ESUBSET \\ & ty.2Erealx.2Ereal)\ V0s)\ V1t)) \wedge (\forall V2x \in ty.2Erealx.2Ereal. \\ & (((p\ (ap\ (ap\ c.2Ereal\_topology.2Elimit\_point\_of\ V2x)\ V0s)) \wedge \\ & (p\ (ap\ (ap\ (c.2Ebool.2EIN\ ty.2Erealx.2Ereal)\ V2x)\ V1t))) \Rightarrow (p\ ( \\ & ap\ (ap\ (c.2Ebool.2EIN\ ty.2Erealx.2Ereal)\ V2x)\ V0s)))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & (\forall V0s \in (2^{ty.2Erealx.2Ereal}). (\forall V1a \in ty.2Erealx.2Ereal. \\ & ((p\ (ap\ (c.2Ereal\_topology.2Etrivial\_limit\ ty.2Erealx.2Ereal) \\ & (ap\ (ap\ (c.2Ereal\_topology.2Ewithin\ ty.2Erealx.2Ereal)\ (ap \\ & c.2Ereal\_topology.2Eat\ V1a))\ V0s))) \Leftrightarrow (\neg (p\ (ap\ (ap\ c.2Ereal\_topology.2Elimit\_point\_of \\ & V1a)\ V0s)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (ty.2Erealx.2Ereal^{A.27a}). \\ & (\forall V1l \in ty.2Erealx.2Ereal. (\forall V2net \in (ty.2Ereal\_topology.2Eenet \\ & A.27a). ((p\ (ap\ (ap\ (ap\ (c.2Ereal\_topology.2E.2D.2D.3E\ A.27a) \\ & V0f)\ V1l)\ V2net)) \Leftrightarrow ((p\ (ap\ (c.2Ereal\_topology.2Etrivial\_limit \\ & A.27a)\ V2net)) \vee (\forall V3e \in ty.2Erealx.2Ereal. ((p\ (ap\ (ap\ c.2Erealx.2Ereal\_lt \\ & (ap\ c.2Ereal.2Ereal\_of\_num\ c.2Enum.2E0))\ V3e)) \Rightarrow (\exists V4y \in \\ & A.27a. ((\exists V5x \in A.27a. (p\ (ap\ (ap\ (ap\ (c.2Ereal\_topology.2Eenetord \\ & A.27a)\ V2net)\ V5x)\ V4y))) \wedge (\forall V6x \in A.27a. ((p\ (ap\ (ap\ (ap\ (c.2Ereal\_topology.2Eenetord \\ & A.27a)\ V2net)\ V6x)\ V4y))) \Rightarrow (p\ (ap\ (ap\ c.2Erealx.2Ereal\_lt\ (ap\ c.2Ereal\_topology.2EDist \\ & (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealx.2Ereal\ ty.2Erealx.2Ereal) \\ & (ap\ V0f\ V6x))\ V1l)))\ V3e)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1s \in \\
& (\mathcal{2}^{ty\_2Erealax\_2Ereal}).((p (ap (ap c\_2Ereal\_topology\_2Econtinuous\_on \\
& V0f) V1s)) \Leftrightarrow (\forall V2x \in ty\_2Erealax\_2Ereal.((p (ap (ap (c\_2Ebool\_2EIN \\
& ty\_2Erealax\_2Ereal) V2x) V1s)) \Rightarrow (p (ap (ap (ap (c\_2Ereal\_topology\_2E\_2D\_2D\_3E \\
& ty\_2Erealax\_2Ereal) V0f) (ap V0f V2x)) (ap (ap (c\_2Ereal\_topology\_2Ewithin \\
& ty\_2Erealax\_2Ereal) (ap c\_2Ereal\_topology\_2Eat V2x)) V1s))))))))) \\
& \hspace{15em} (48)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1l \in \\
& ty\_2Erealax\_2Ereal.(\forall V2x \in ty\_2Erealax\_2Ereal.(\forall V3s \in \\
& (\mathcal{2}^{ty\_2Erealax\_2Ereal}).(\forall V4t \in (\mathcal{2}^{ty\_2Erealax\_2Ereal}). \\
& ((p (ap (ap (ap (c\_2Ereal\_topology\_2E\_2D\_2D\_3E ty\_2Erealax\_2Ereal) \\
& V0f) V1l) (ap (ap (c\_2Ereal\_topology\_2Ewithin ty\_2Erealax\_2Ereal) \\
& (ap c\_2Ereal\_topology\_2Eat V2x)) (ap (ap (c\_2Epred\_set\_2EUNION \\
& ty\_2Erealax\_2Ereal) V3s) V4t))) \Leftrightarrow ((p (ap (ap (ap (c\_2Ereal\_topology\_2E\_2D\_2D\_3E \\
& ty\_2Erealax\_2Ereal) V0f) V1l) (ap (ap (c\_2Ereal\_topology\_2Ewithin \\
& ty\_2Erealax\_2Ereal) (ap c\_2Ereal\_topology\_2Eat V2x)) V3s))) \wedge \\
& (p (ap (ap (ap (c\_2Ereal\_topology\_2E\_2D\_2D\_3E ty\_2Erealax\_2Ereal) \\
& V0f) V1l) (ap (ap (c\_2Ereal\_topology\_2Ewithin ty\_2Erealax\_2Ereal) \\
& (ap c\_2Ereal\_topology\_2Eat V2x)) V4t))))))))) \\
& \hspace{15em} (49)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \hspace{10em} (50)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \hspace{10em} (51)$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \hspace{10em} (52)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \hspace{10em} (53)
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \hspace{10em} (54)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))) \\
& \hspace{15em} (55)
\end{aligned}$$



Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ( \\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{59}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0t \in (2^{ty\_2Erealax\_2Ereal}). (\forall V1f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). \\
& (\forall V2s \in (2^{ty\_2Erealax\_2Ereal}). (((p (ap (ap (c\_2Etopology\_2Eclosed\_in \\
& ty\_2Erealax\_2Ereal) (ap (ap (c\_2Ereal\_topology\_2Esubtopology \\
& ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Eeuclidean) (ap (ap \\
& (c\_2Epred\_set\_2EUNION ty\_2Erealax\_2Ereal) V2s) V0t))) V2s)) \wedge \\
& ((p (ap (ap (c\_2Etopology\_2Eclosed\_in ty\_2Erealax\_2Ereal) ( \\
& ap (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\
& c\_2Ereal\_topology\_2Eeuclidean) (ap (ap (c\_2Epred\_set\_2EUNION \\
& ty\_2Erealax\_2Ereal) V2s) V0t))) V0t)) \wedge ((p (ap (ap c\_2Ereal\_topology\_2Econtinuous\_on \\
& V1f) V2s)) \wedge (p (ap (ap c\_2Ereal\_topology\_2Econtinuous\_on V1f) \\
& V0t)))))) \Rightarrow (p (ap (ap c\_2Ereal\_topology\_2Econtinuous\_on V1f) \\
& (ap (ap (c\_2Epred\_set\_2EUNION ty\_2Erealax\_2Ereal) V2s) V0t))))))
\end{aligned}$$