

thm_2Ereal__topology_2ECONTINUOUS__TRANSFORM__WITHI (TMFnH3GuoH3FjDs6HkaCL9QiSwRW8An6tcs)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** *(the* $(\lambda x.x \in A \wedge p x)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a P))))$

Definition 4 We define $c_2Ebool_2E_5C$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}) P))))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (2)$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2E_21 2) (ap (c_2Emin_2E_3D (2^{A_27a}) V0x) V1y))$

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty ty_2Erealx_2Ereal \quad (3)$$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealx_2Ereal^{(ty_2Epair_2Eprod ty_2Erealx_2Ereal ty_2Erealx_2Ereal)}) \quad (4)$$

Definition 9 We define c_Ebool_EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Let $c_Enum_EZERO_REP : \iota$ be given. Assume the following.

$$c_Enum_EZERO_REP \in \omega \tag{5}$$

Let $ty_Enum_Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_Enum_Enum \tag{6}$$

Let $c_Enum_EABS_num : \iota$ be given. Assume the following.

$$c_Enum_EABS_num \in (ty_Enum_Enum^{\omega}) \tag{7}$$

Definition 10 We define c_Enum_E0 to be $(ap\ c_Enum_EABS_num\ c_Enum_EZERO_REP)$.

Let $c_Ereal_Ereal_of_num : \iota$ be given. Assume the following.

$$c_Ereal_Ereal_of_num \in (ty_Erealax_Ereal^{ty_Enum_Enum}) \tag{8}$$

Let $ty_Ehreal_Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_Ehreal_Ehreal \tag{9}$$

Let $c_Erealax_Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_Ereal_REP_CLASS \in ((2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})^{ty_Erealax_Ereal}) \tag{10}$$

Definition 11 We define $c_Erealax_Ereal_REP$ to be $\lambda V0a \in ty_Erealax_Ereal. (ap\ (c_Emin_E_40\ t))$

Let $c_Erealax_Etreall_lt : \iota$ be given. Assume the following.

$$c_Erealax_Etreall_lt \in ((2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal)}) \tag{11}$$

Definition 12 We define $c_Erealax_Ereal_lt$ to be $\lambda V0T1 \in ty_Erealax_Ereal. \lambda V1T2 \in ty_Erealax_Ereal.$

Let $ty_Ereal_topology_Enet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_Ereal_topology_Enet\ A0) \tag{12}$$

Let $c_Ereal_topology_Enetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_Ereal_topology_Enetord\ A_27a \in ((2^{A_27a})^{A_27a})^{(ty_Ereal_topology_Enet\ A_27a)} \tag{13}$$

Definition 13 We define c_Ebool_EF to be $(ap\ (c_Ebool_E_21\ 2)\ (\lambda V0t \in 2. V0t))$.

Definition 14 We define $c_Ebool_E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_Emin_E_3D_3D_3E\ V0t)\ c_Ebool_E_21))$

Definition 15 We define $c_Ebool_E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_Ebool_E_21\ 2)\ (\lambda V2t \in 2. V2t))))$

Definition 16 We define $c_2Ereal_topology_2Etrivial_limit$ to be $\lambda A_27a : \iota.\lambda V0net \in (ty_2Ereal_topology_2E$

Definition 17 We define $c_2Ereal_topology_2Eeventually$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1net \in (ty_2E$

Definition 18 We define $c_2Ereal_topology_2E_2D_2D_3E$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal^A$

Definition 19 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2E$

Let $c_2Ereal_topology_2Emk_net : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ereal_topology_2Emk_net \\ A_27a \in ((ty_2Ereal_topology_2Enet\ A_27a)^{(2^{A_27a})^{A_27a}}) \end{aligned} \quad (14)$$

Definition 20 We define $c_2Ereal_topology_2Eat$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Ereal_topology_2E$

Definition 21 We define $c_2Ereal_topology_2Ewithin$ to be $\lambda A_27a : \iota.\lambda V0net \in (ty_2Ereal_topology_2E$

Definition 22 We define $c_2Ereal_topology_2Eenlimit$ to be $\lambda A_27a : \iota.\lambda V0net \in (ty_2Ereal_topology_2E$

Definition 23 We define $c_2Ereal_topology_2Econtinuous$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal^A$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ (p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} ((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (19)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (21)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \wedge (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C)))))) \quad (22)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V1B) \wedge (p V2C) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (23)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (24)$$

Assume the following.

$$2.(((p V0x) \Leftrightarrow (p V1x_{.27}) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27}))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (25)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap\ c_2Ereal_topology_2EDist (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0x) V0x)) = (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))) \quad (26)$$

Assume the following.

$$(\forall V0l \in ty_2Erealax_2Ereal.(\forall V1f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V2g \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V3x \in ty_2Erealax_2Ereal.(\forall V4s \in (2^{ty_2Erealax_2Ereal}).(\forall V5d \in ty_2Erealax_2Ereal.(((p (ap (ap (c_2Erealax_2Ereal_It (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0) V5d)) \wedge ((\forall V6x_{.27} \in ty_2Erealax_2Ereal.(((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V6x_{.27}) V4s)) \wedge ((p (ap (ap (c_2Erealax_2Ereal_It (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) (ap\ c_2Ereal_topology_2EDist (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V6x_{.27}) V3x)))) \wedge (p (ap (ap (c_2Erealax_2Ereal_It (ap\ c_2Ereal_topology_2EDist (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V6x_{.27}) V3x)) V5d)))) \Rightarrow ((ap V1f V6x_{.27}) = (ap V2g V6x_{.27}))) \wedge (p (ap (ap (ap (c_2Ereal_topology_2E_2D_2D_3E ty_2Erealax_2Ereal) V1f) V0l) (ap (ap (c_2Ereal_topology_2Ewithin ty_2Erealax_2Ereal) (ap\ c_2Ereal_topology_2Eat V3x)) V4s)))))) \Rightarrow (p (ap (ap (ap (c_2Ereal_topology_2E_2D_2D_3E ty_2Erealax_2Ereal) V2g) V0l) (ap (ap (c_2Ereal_topology_2Ewithin ty_2Erealax_2Ereal) (ap\ c_2Ereal_topology_2Eat V3x)) V4s)))))))))) \quad (27)$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). \\
& (\forall V2x \in ty_2Erealax_2Ereal.((p (ap (ap (c_2Ereal_topology_2Econtinuous \\
& ty_2Erealax_2Ereal) V1f) (ap (ap (c_2Ereal_topology_2Ewithin \\
& ty_2Erealax_2Ereal) (ap c_2Ereal_topology_2Eat V2x)) V0s))) \Leftrightarrow \\
& (p (ap (ap (ap (c_2Ereal_topology_2E_2D_2D_3E ty_2Erealax_2Ereal) \\
& V1f) (ap V1f V2x)) (ap (ap (c_2Ereal_topology_2Ewithin ty_2Erealax_2Ereal) \\
& (ap c_2Ereal_topology_2Eat V2x)) V0s)))))))))
\end{aligned} \tag{28}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{29}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{32}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r)))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{36}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\neg(p V1q) \vee ((p V2r) \vee \neg(p V0p)))))))) \quad (37)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (\neg(p V1q) \vee \neg(p V0p)))))) \quad (38)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (39)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow \neg(p V1q)))) \quad (40)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V0p)))) \quad (41)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V1q)))) \quad (42)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (43)$$

Theorem 1

$$\begin{aligned} & (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V1g \in \\ & (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V2s \in (2^{ty_2Erealax_2Ereal}). \\ & (\forall V3x \in ty_2Erealax_2Ereal. (\forall V4d \in ty_2Erealax_2Ereal. \\ & (((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\ & c_2Enum_2E0)) V4d)) \wedge ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\ & V3x) V2s)) \wedge ((\forall V5x_27 \in ty_2Erealax_2Ereal. (((p (ap (ap \\ & (c_2Ebool_2EIN ty_2Erealax_2Ereal) V5x_27) V2s)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt \\ & (ap c_2Ereal_topology_2EDist (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\ & ty_2Erealax_2Ereal) V5x_27) V3x))) V4d))) \Rightarrow ((ap V0f V5x_27) = (\\ & ap V1g V5x_27)))) \wedge (p (ap (ap (c_2Ereal_topology_2Econtinuous \\ & ty_2Erealax_2Ereal) V0f) (ap (ap (c_2Ereal_topology_2Ewithin \\ & ty_2Erealax_2Ereal) (ap c_2Ereal_topology_2Eat V3x)) V2s)))))) \Rightarrow \\ & (p (ap (ap (c_2Ereal_topology_2Econtinuous ty_2Erealax_2Ereal) \\ & V1g) (ap (ap (c_2Ereal_topology_2Ewithin ty_2Erealax_2Ereal) \\ & (ap c_2Ereal_topology_2Eat V3x)) V2s)))))) \end{aligned}$$