# thm_2Ereal__topology_2ECONTINUOUS__TRANSFORM__WITHI (TMY5M29YBbxdQQsDq4MwB4RSxneSbHAshjF)

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**Definition 1** *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

**Definition 2** *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

**Definition 3** *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

**Definition 4** *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

**Definition 5** *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* $\iota$.

**Definition 6** *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

**Definition 7** *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Let $ty\_2Ereal\_\_topology\_2Enet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ereal\_\_topology\_2Enet\ A0) \qquad (1)$$

Let $c\_2Ereal\_\_topology\_2Enetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ereal\_\_topology\_2Enetord\ A\_27a \in (((2^{A\_27a})^{A\_27a})^{(ty\_2Ereal\_\_topology\_2Enet\ A\_27a)}) \qquad (2)$$

**Definition 8** *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

**Definition 9** *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

**Definition 10** *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

**Definition 11** *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

**Definition 12** *We define* c_2Ereal__topology_2Etrivial__limit *to be* $\lambda A\_27a:\iota.\lambda V0net \in (ty\_2Ereal\_\_topology$

**Definition 13** *We define* c_2Ereal__topology_2Eeventually *to be* $\lambda A\_27a:\iota.\lambda V0p \in (2^{A\_27a}).\lambda V1net \in (ty\_2$

Let $ty\_2Erealax\_2Ereal:\iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Let $ty\_2Epair\_2Eprod:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0\Rightarrow\forall A1.nonempty\ A1\Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{4}$$

Let $c\_2Epair\_2EABS\_\_prod:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{5}$$

**Definition 14** *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Ereal\_\_topology\_2EDist:\iota$ be given. Assume the following.

$$c\_2Ereal\_\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \tag{6}$$

Let $ty\_2Ehreal\_2Ehreal:\iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{7}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS:\iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{8}$$

**Definition 15** *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(t$

Let $c\_2Erealax\_2Etreal\_\_lt:\iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehr} \tag{9}$$

**Definition 16** *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Let $c\_2Enum\_2EZERO\_\_REP:\iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{10}$$

Let $ty\_2Enum\_2Enum:\iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{11}$$

Let $c\_2Enum\_2EABS\_\_num:\iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{12}$$

**Definition 17** *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{13}$$

**Definition 18** *We define* c_2Ereal__topology_2E_2D_2D_3E *to be* $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Erealax\_2Ereal^{A\text{-}}$

**Definition 19** *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Let $c\_2Ereal\_\_topology\_2Emk\_\_net : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ereal\_\_topology\_2Emk\_\_net\ A\_27a \in ((ty\_2Ereal\_\_topology\_2Enet\ A\_27a)^{((2^{A\_27a})^{A\_27a})}) \tag{14}$$

**Definition 20** *We define* c_2Ereal__topology_2Eat *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Ereal\_\_topolo$

**Definition 21** *We define* c_2Ereal__topology_2Ewithin *to be* $\lambda A\_27a : \iota.\lambda V0net \in (ty\_2Ereal\_\_topology\_2En$

**Definition 22** *We define* c_2Ereal__topology_2Enetlimit *to be* $\lambda A\_27a : \iota.\lambda V0net \in (ty\_2Ereal\_\_topology\_2E$

**Definition 23** *We define* c_2Ereal__topology_2Econtinuous *to be* $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Erealax\_2Ereal^{A\text{-}}$

Assume the following.

$$True \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{17}$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1l \in ty\_2Erealax\_2Ereal.(\forall V2a \in ty\_2Erealax\_2Ereal.(\forall V3s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V4t \in (2^{ty\_2Erealax\_2Ereal}).(((p\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Eeventually}\ ty\_2Erealax\_2Ereal)\ (\lambda V5x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V5x)\ V4t))\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V5x)\ V3s))))\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Eat}\ V2a))) \wedge (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2E\_2D\_2D\_3E}\ ty\_2Erealax\_2Ereal)\ V0f)\ V1l)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Ewithin}\ ty\_2Erealax\_2Ereal)\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Eat}\ V2a))\ V3s)))) \Rightarrow (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2E\_2D\_2D\_3E}\ ty\_2Erealax\_2Ereal)\ V0f)\ V1l)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Ewithin}\ ty\_2Erealax\_2Ereal)\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Eat}\ V2a))\ V4t))))))))) \tag{18}$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2x \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Econtinuous}\ ty\_2Erealax\_2Ereal)\ V1f)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Ewithin}\ ty\_2Erealax\_2Ereal)\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Eat}\ V2x))\ V0s)))\Leftrightarrow (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2E\_2D\_2D\_3E}\ ty\_2Erealax\_2Ereal)\ V1f)\ (ap\ V1f\ V2x))\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Ewithin}\ ty\_2Erealax\_2Ereal)\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Eat}\ V2x))\ V0s))))))) \tag{19}$$

**Theorem 1**

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1a \in ty\_2Erealax\_2Ereal.(\forall V2s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V3t \in (2^{ty\_2Erealax\_2Ereal}).(((p\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Eeventually}\ ty\_2Erealax\_2Ereal)\ (\lambda V4x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V4x)\ V3t))\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V4x)\ V2s))))\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Eat}\ V1a)))\wedge(p\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Econtinuous}\ ty\_2Erealax\_2Ereal)\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Ewithin}\ ty\_2Erealax\_2Ereal)\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Eat}\ V1a))\ V2s))))\Rightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Econtinuous}\ ty\_2Erealax\_2Ereal)\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Ewithin}\ ty\_2Erealax\_2Ereal)\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Eat}\ V1a))\ V3t))))))))$$