

# thm\_2Ereal\_\_topology\_2EDENSE\_\_OPEN\_\_INTER (TMXQu2C6MJRYeccgx9TKcgKY5oXJkysSywz)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 4** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 6** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (1)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (2)$$

**Definition 7** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epair\_2EABS\_prod$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \quad (3)$$

**Definition 8** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Epair\_2EABS\_prod$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (4)$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (5)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (6)$$

**Definition 9** We define  $c\_2Emin\_2E40$  to be  $\lambda A.\lambda P \in 2^A$ . **if**  $(\exists x \in A.p (ap\ P\ x))$  **then** (the  $(\lambda x.x \in A \wedge p\ x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 10** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E40\ a))$

Let  $c\_2Erealax\_2Etrealmul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealmul \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (7)$$

**Definition 11** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 12** We define  $c\_2Ebool\_2E21$  to be  $(ap\ (c\_2Ebool\_2E21\ 2))\ (\lambda V0t \in 2.V0t)$ .

**Definition 13** We define  $c\_2Ebool\_2E7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E21))$

**Definition 14** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

**Definition 15** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.(ap\ c\_2Emin\_2E3D\_3D\_3E\ t1\ t2))))$

**Definition 16** We define  $c\_2Ereal\_2Emin$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

Let  $c\_2Erealax\_2Etrealmul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealmul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (8)$$

Let  $c\_2Erealax\_2Ereal\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (9)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (10)$$

**Definition 17** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 18** We define  $c\_Erealax\_Ereal\_mul$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.\lambda V1T2 \in ty\_Erealax\_Ereal$ .  
Let  $c\_Enum\_EZERO\_REP : \iota$  be given. Assume the following.

$$c\_Enum\_EZERO\_REP \in \omega \tag{11}$$

Let  $ty\_Enum\_Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_Enum\_Enum \tag{12}$$

Let  $c\_Enum\_EABS\_num : \iota$  be given. Assume the following.

$$c\_Enum\_EABS\_num \in (ty\_Enum\_Enum^{\omega}) \tag{13}$$

**Definition 19** We define  $c\_Enum\_E0$  to be  $(ap\ c\_Enum\_EABS\_num\ c\_Enum\_EZERO\_REP)$ .

**Definition 20** We define  $c\_Earithmetic\_EZERO$  to be  $c\_Enum\_E0$ .

Let  $c\_Enum\_EREP\_num : \iota$  be given. Assume the following.

$$c\_Enum\_EREP\_num \in (\omega^{ty\_Enum\_Enum}) \tag{14}$$

Let  $c\_Enum\_ESUC\_REP : \iota$  be given. Assume the following.

$$c\_Enum\_ESUC\_REP \in (\omega^{\omega}) \tag{15}$$

**Definition 21** We define  $c\_Enum\_ESUC$  to be  $\lambda V0m \in ty\_Enum\_Enum.(ap\ c\_Enum\_EABS\_num\ m)$ .

Let  $c\_Earithmetic\_E\_2B : \iota$  be given. Assume the following.

$$c\_Earithmetic\_E\_2B \in ((ty\_Enum\_Enum^{ty\_Enum\_Enum})^{ty\_Enum\_Enum}) \tag{16}$$

**Definition 22** We define  $c\_Earithmetic\_EBIT2$  to be  $\lambda V0n \in ty\_Enum\_Enum.(ap\ (ap\ c\_Earithmetic\_E\_2B\ n))$ .

**Definition 23** We define  $c\_Earithmetic\_ENUMERAL$  to be  $\lambda V0x \in ty\_Enum\_Enum.V0x$ .

Let  $c\_Erealax\_Etrealm\_inv : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealm\_inv \in ((ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)}) \tag{17}$$

**Definition 24** We define  $c\_Erealax\_Einv$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.(ap\ c\_Erealax\_Ereal\_ABS\ T1)$ .

**Definition 25** We define  $c\_Ereal\_E\_2F$  to be  $\lambda V0x \in ty\_Erealax\_Ereal.\lambda V1y \in ty\_Erealax\_Ereal.(ap\ c\_Erealax\_Ereal\_ABS\ x\ y)$ .

Let  $c\_Erealax\_Etrealm\_add : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealm\_add \in (((ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)})^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)}) \tag{18}$$

**Definition 26** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$ . Let  $c\_2Ereal\_topology\_2EDist : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \quad (19)$$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum}) \quad (20)$$

**Definition 27** We define  $c\_2Ebool\_2E3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E40$

**Definition 28** We define  $c\_2Ereal\_topology\_2EOpen$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap\ (c\_2Ebool\_2E2$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (21)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^A-27a)})}) \quad (22)$$

**Definition 29** We define  $c\_2Ereal\_topology\_2Eeuclidean$  to be  $(ap\ (c\_2Etopology\_2Etopology\ ty\_2Erealax\_2Ereal$

Let  $c\_2Etopology\_2Eopen\_in : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Eopen\_in\ A\_27a \in ((2^{(2^A-27a)})^{(ty\_2Etopology\_2Etopology\ A\_27a)}) \quad (23)$$

**Definition 30** We define  $c\_2Ereal\_topology\_2Esubtopology$  to be  $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology$

**Definition 31** We define  $c\_2Ereal\_topology\_2Elimit\_point\_of$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1s \in ($

**Definition 32** We define  $c\_2Ebool\_2E5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V2t \in$

**Definition 33** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ (c$

**Definition 34** We define  $c\_2Ereal\_topology\_2Eclosure$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap\ (ap\ (c\_2Epred$

**Definition 35** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ ($

Assume the following.

$$True \quad (24)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (25)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (29) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee \\ & (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (30) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (31) \end{aligned}$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \quad (32) \end{aligned}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (33)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ & p\ V0t)))))) \quad (35) \end{aligned}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))))))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \wedge (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C))))))))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A))))))))) \quad (40)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))))) \quad (41)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.(((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27))))))))) \quad (42)$$

Assume the following.

$$(\forall V0r \in 2.(\forall V1p \in 2.(\forall V2q \in 2.(((p V1p) \wedge (p V2q)) \Leftrightarrow ((p V2q) \wedge (p V1p))) \wedge (((p V1p) \wedge (p V2q)) \wedge (p V0r)) \Leftrightarrow ((p V1p) \wedge ((p V2q) \wedge (p V0r)))) \wedge (((p V1p) \wedge (p V2q) \wedge (p V0r)) \Leftrightarrow ((p V2q) \wedge ((p V1p) \wedge (p V0r)))) \wedge (((p V1p) \wedge (p V1p)) \Leftrightarrow (p V1p)) \wedge (((p V1p) \wedge (p V1p) \wedge (p V2q)) \Leftrightarrow ((p V1p) \wedge (p V2q))))))))) \quad (43)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(\forall V2u \in (2^{A\_27a}).(((p (ap (ap (c\_2Epred\_set\_2ESUBSET A\_27a) V0s) V1t)) \wedge (p (ap (ap (c\_2Epred\_set\_2ESUBSET A\_27a) V1t) V2u)))) \Rightarrow (p (ap (ap (c\_2Epred\_set\_2ESUBSET A\_27a) V0s) V2u))))))))) \quad (44)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A-27a}).(\forall V1t \in \\ (2^{A-27a}).(\forall V2x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a) \\ V2x)\ (ap\ (ap\ (c.2Epred\_set.2EINTER\ A.27a)\ V0s)\ V1t)))) \Leftrightarrow ((p\ (ap\ (45) \\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c.2Ebool.2EIN \\ A.27a)\ V2x)\ V1t)))))) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A-27a}).(\forall V1t \in \\ (2^{A-27a}).((ap\ (ap\ (c.2Epred\_set.2EINTER\ A.27a)\ V0s)\ V1t) = ( \\ ap\ (ap\ (c.2Epred\_set.2EINTER\ A.27a)\ V1t)\ V0s)))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0s \in (2^{A-27a}).(\forall V1t \in \\ (2^{A-27a}).(p\ (ap\ (ap\ (c.2Epred\_set.2ESUBSET\ A.27a)\ (ap\ (ap\ (c.2Epred\_set.2EINTER \\ A.27a)\ V0s)\ V1t))\ V0s)))) \wedge (\forall V2s \in (2^{A-27a}).(\forall V3t \in \\ (2^{A-27a}).(p\ (ap\ (ap\ (c.2Epred\_set.2ESUBSET\ A.27a)\ (ap\ (ap\ (c.2Epred\_set.2EINTER \\ A.27a)\ V3t)\ V2s))\ V2s)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\ (\forall V2z \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ c.2Ereal\_2Ereal\_lte \\ V0x)\ V1y)) \wedge (p\ (ap\ (ap\ c.2Erealax\_2Ereal\_lt\ V1y)\ V2z))) \Rightarrow (p\ (ap \\ (ap\ c.2Erealax\_2Ereal\_lt\ V0x)\ V2z)))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} (\forall V0w \in ty\_2Erealax\_2Ereal.(\forall V1x \in ty\_2Erealax\_2Ereal. \\ (\forall V2y \in ty\_2Erealax\_2Ereal.(\forall V3z \in ty\_2Erealax\_2Ereal. \\ (((p\ (ap\ (ap\ c.2Erealax\_2Ereal\_lt\ V0w)\ V1x)) \wedge (p\ (ap\ (ap\ c.2Erealax\_2Ereal\_lt \\ V2y)\ V3z)))) \Rightarrow (p\ (ap\ (ap\ c.2Erealax\_2Ereal\_lt\ (ap\ (ap\ c.2Erealax\_2Ereal\_add \\ V0w)\ V2y))\ (ap\ (ap\ c.2Erealax\_2Ereal\_add\ V1x)\ V3z)))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} (\forall V0d \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ c.2Erealax\_2Ereal\_lt \\ (ap\ c.2Ereal\_2Ereal\_of\_num\ c.2Enum.2E0))\ (ap\ (ap\ c.2Ereal.2E\_2F \\ V0d)\ (ap\ c.2Ereal\_2Ereal\_of\_num\ (ap\ c.2Earithmetic.2ENUMERAL \\ (ap\ c.2Earithmetic.2EBIT2\ c.2Earithmetic.2EZERO)))))) \Leftrightarrow (p\ ( \\ ap\ (ap\ c.2Erealax\_2Ereal\_lt\ (ap\ c.2Ereal\_2Ereal\_of\_num\ c.2Enum.2E0)) \\ V0d)))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_add \\
& (ap (ap c\_2Ereal\_2E\_2F V0x) (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))))) (ap (ap \\
& c\_2Ereal\_2E\_2F V0x) (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))))) = V0x)) \\
& \hspace{15em} (51)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& V2z) (ap (ap c\_2Ereal\_2Emin V0x) V1y))) \Leftrightarrow ((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& V2z) V0x)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt V2z) V1y))))))) \\
& \hspace{15em} (52)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0e \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) (ap (ap c\_2Ereal\_2E\_2F \\
& V0e) (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))))) \Leftrightarrow (p ( \\
& ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \\
& V0e)))) \wedge ((\forall V1e \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_add \\
& (ap (ap c\_2Ereal\_2E\_2F V1e) (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))))) (ap (ap \\
& c\_2Ereal\_2E\_2F V1e) (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))))) = V1e)) \wedge \\
& (\forall V2e \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL ( \\
& ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))) (ap (ap c\_2Ereal\_2E\_2F \\
& V2e) (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))))) = V2e)))))) \\
& \hspace{15em} (53)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& (ap c\_2Ereal\_topology\_2EDist (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& ty\_2Erealax\_2Ereal) V0x) V2z))) (ap (ap c\_2Erealax\_2Ereal\_add \\
& (ap c\_2Ereal\_topology\_2EDist (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& ty\_2Erealax\_2Ereal) V0x) V1y))) (ap c\_2Ereal\_topology\_2EDist \\
& (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& V1y) V2z)))))))))) \\
& \hspace{15em} (54)
\end{aligned}$$



Assume the following.

$$\begin{aligned}
& (\forall V0u \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1s \in (2^{ty\_2Erealax\_2Ereal}). \\
& ((p (ap (ap (ap (c\_2Etopology\_2Eopen\_in ty\_2Erealax\_2Ereal) (ap \\
& (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\
& c\_2Ereal\_topology\_2Eeuclidean) V0u)) V1s)) \Leftrightarrow ((p (ap (ap (c\_2Epred\_set\_2ESUBSET \\
& ty\_2Erealax\_2Ereal) V1s) V0u)) \wedge (\forall V2x \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V2x) V1s)) \Rightarrow (\exists V3e \in \\
& ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) V3e)) \wedge (\forall V4x\_27 \in ty\_2Erealax\_2Ereal. (( \\
& (p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V4x\_27) V0u)) \wedge ( \\
& p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_topology\_2EDist \\
& (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& V4x\_27) V2x))) V3e))) \Rightarrow (p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\
& V4x\_27) V1s))))))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1s \in (2^{ty\_2Erealax\_2Ereal}). \\
& ((p (ap (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V0x) (ap c\_2Ereal\_topology\_2Eclosure \\
& V1s))) \Leftrightarrow (\forall V2e \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V2e)) \Rightarrow (\exists V3y \in \\
& ty\_2Erealax\_2Ereal. ((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\
& V3y) V1s)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_topology\_2EDist \\
& (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& V3y) V0x))) V2e))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1t \in (2^{ty\_2Erealax\_2Ereal}). \\
& ((p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) V0s) \\
& V1t)) \Rightarrow (p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) \\
& (ap c\_2Ereal\_topology\_2Eclosure V0s)) (ap c\_2Ereal\_topology\_2Eclosure \\
& V1t))))))
\end{aligned} \tag{57}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{58}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{60}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (61)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (62)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg( \\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (63) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (64) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (65) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ( \\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (66) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (67) \end{aligned}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (68)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (69)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1t \in (2^{ty\_2Erealax\_2Ereal}). \\ & (\forall V2u \in (2^{ty\_2Erealax\_2Ereal}).((((p (ap (ap (c\_2Etopology\_2Eopen\_in \\ & ty\_2Erealax\_2Ereal) (ap (ap (c\_2Ereal\_topology\_2Esubtopology \\ & ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Eeuclidean) V2u)) \\ & V0s)) \wedge (p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) \\ & V1t) V2u))) \vee ((p (ap (ap (c\_2Etopology\_2Eopen\_in ty\_2Erealax\_2Ereal) \\ & (ap (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\ & c\_2Ereal\_topology\_2Eeuclidean) V2u)) V1t)) \wedge (p (ap (ap (c\_2Epred\_set\_2ESUBSET \\ & ty\_2Erealax\_2Ereal) V0s) V2u)))) \Rightarrow ((p (ap (ap (c\_2Epred\_set\_2ESUBSET \\ & ty\_2Erealax\_2Ereal) V2u) (ap c\_2Ereal\_topology\_2Eclosure ( \\ & ap (ap (c\_2Epred\_set\_2EINTER ty\_2Erealax\_2Ereal) V0s) V1t)))) \Leftrightarrow \\ & ((p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) V2u) \\ & (ap c\_2Ereal\_topology\_2Eclosure V0s))) \wedge (p (ap (ap (c\_2Epred\_set\_2ESUBSET \\ & ty\_2Erealax\_2Ereal) V2u) (ap c\_2Ereal\_topology\_2Eclosure V1t)))))))))) \end{aligned}$$