

# thm\_2Ereal\_\_topology\_2EDISTANCE\_\_ATTAINS\_\_SUP (TMNBvZwSR3mRSTrFapzRDkPdgstn4u1Jega)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 3** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 4** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (ap\ c\_2Enum\_2EREP\_num\ c\_2Enum\_2ESUC\_REP))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})ty\_2Enum\_2Enum) \quad (6)$$

**Definition 7** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B))$

**Definition 8** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (7)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (8)$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (9)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})ty\_2Erealax\_2Ereal) \quad (10)$$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A$ . **if**  $(\exists x \in A.p (ap\ P\ x))$  **then** (the  $(\lambda x.x \in A \wedge p)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 10** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E\_40))$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (11)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (12)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (13)$$

**Definition 11** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 12** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_neg)$

Let  $c\_2Erealax\_2Etreall\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)))(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal) \quad (14)$$

**Definition 13** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Etreall\_mul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)))(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal) \quad (15)$$

**Definition 14** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 15** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 16** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21\ 2) (\lambda V2t \in 2.))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (16)$$

**Definition 17** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epair\_2EABS\_prod$

Let  $c\_2Ereal\_topology\_2EDist : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \quad (17)$$

**Definition 18** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum}) \quad (18)$$

Let  $c\_2Erealax\_2Etreall\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)) \quad (19)$$

**Definition 19** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 20** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21\ 2) (\lambda V0t \in 2.V0t))$ .

**Definition 21** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_21\ 2))$

**Definition 22** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 23** We define  $c\_Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

**Definition 24** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal. (ap (ap (ap (c\_2Ebool\_2ECOND$

**Definition 25** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. (\lambda V1f \in (2^{A\_27a}). (ap V1f V0x))$

**Definition 26** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap V0P (ap (c\_2Emin\_2E\_40$

**Definition 27** We define  $c\_2Ereal\_topology\_2Econtinuous\_on$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Ereal$

**Definition 28** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. c\_2Ebool\_2EF)$ .

**Definition 29** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 30** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 31** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in$

**Definition 32** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

Let  $ty\_2Ereal\_topology\_2Eenet : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Ereal\_topology\_2Eenet A0) \quad (20)$$

Let  $c\_2Ereal\_topology\_2Emk\_net : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty A\_27a \Rightarrow c\_2Ereal\_topology\_2Emk\_net \\ A\_27a \in ((ty\_2Ereal\_topology\_2Eenet A\_27a)^{(2^{A\_27a})^{A\_27a}}) \end{aligned} \quad (21)$$

**Definition 33** We define  $c\_2Ereal\_topology\_2Esequentially$  to be  $(ap (c\_2Ereal\_topology\_2Emk\_net ty\_2E$

**Definition 34** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V0f \in (A\_27b^{A\_27c}). \lambda V1$

Let  $c\_2Ereal\_topology\_2Eenetord : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Ereal\_topology\_2Eenetord A\_27a \in ((2^{A\_27a})^{A\_27a})^{(ty\_2Ereal\_topology\_2Eenet A\_27a)} \quad (22)$$

**Definition 35** We define  $c\_2Ereal\_topology\_2Etrivial\_limit$  to be  $\lambda A\_27a : \iota. \lambda V0net \in (ty\_2Ereal\_topology$

**Definition 36** We define  $c\_2Ereal\_topology\_2Eeventually$  to be  $\lambda A\_27a : \iota. \lambda V0p \in (2^{A\_27a}). \lambda V1net \in (ty\_2$

**Definition 37** We define  $c\_2Ereal\_topology\_2E\_2D\_2D\_3E$  to be  $\lambda A\_27a : \iota. \lambda V0f \in (ty\_2Erealax\_2Ereal^A$

**Definition 38** We define  $c\_2Ereal\_topology\_2Ecompact$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}). (ap (c\_2Ebool\_2E$

Assume the following.

$$True \quad (23)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. ((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee \neg(p V0t))) \quad (26)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow \neg(p V0t))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2. (\neg(p V0t) \Rightarrow ((p V0t) \Rightarrow False))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (32)$$

Assume the following.

$$(\forall V0t \in 2. (\neg(\neg(p V0t)) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (33)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (34)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (36)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\neg(\forall V1x \in A\_27a. (p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A\_27a. (\neg(p\ (ap\ V0P\ V2x))))) \quad (37)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\neg(\exists V1x \in A\_27a. (p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a. (\neg(p\ (ap\ V0P\ V2x))))) \quad (38)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A\_27a}). (((p\ V0P) \vee (\exists V2x \in A\_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\exists V3x \in A\_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V3x))))))) \quad (39)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A\_27a}). (((\exists V2x \in A\_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \wedge (\exists V3x \in A\_27a. (p\ (ap\ V1Q\ V3x))))))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C))))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee \neg(p\ V1B)))) \wedge (((\neg(p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A) \wedge \neg(p\ V1B))))) \quad (43)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ & \quad \forall V0P \in ((2^{A.27b})^{A.27a}).((\forall V1x \in A.27a.(\exists V2y \in \\ & A.27b.(p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A.27b^{A.27a}).( \\ & \quad \forall V4x \in A.27a.(p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x)))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\ & ((ap\ (ap\ c\_2Erealax\_2Ereal\_add\ V0x)\ V1y) = (ap\ (ap\ c\_2Erealax\_2Ereal\_add \\ & \quad V1y)\ V0x)))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\ & (\forall V2z \in ty\_2Erealax\_2Ereal.((ap\ (ap\ c\_2Erealax\_2Ereal\_add \\ & V0x)\ (ap\ (ap\ c\_2Erealax\_2Ereal\_add\ V1y)\ V2z)) = (ap\ (ap\ c\_2Erealax\_2Ereal\_add \\ & \quad (ap\ (ap\ c\_2Erealax\_2Ereal\_add\ V0x)\ V1y))\ V2z)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ c\_2Erealax\_2Ereal\_add \\ & (ap\ c\_2Erealax\_2Ereal\_neg\ V0x))\ V0x) = (ap\ c\_2Ereal\_2Ereal\_of\_num \\ & \quad c\_2Enum\_2E0))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ c\_2Erealax\_2Ereal\_mul \\ & (ap\ c\_2Ereal\_2Ereal\_of\_num\ (ap\ c\_2Earithmetic\_2ENUMERAL\ ( \\ & \quad ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))\ V0x) = V0x)) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ c\_2Erealax\_2Ereal\_add \\ & V0x)\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)) = V0x)) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ c\_2Erealax\_2Ereal\_add \\ & V0x)\ (ap\ c\_2Erealax\_2Ereal\_neg\ V0x)) = (ap\ c\_2Ereal\_2Ereal\_of\_num \\ & \quad c\_2Enum\_2E0))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\ & ((ap\ c\_2Erealax\_2Ereal\_neg\ (ap\ (ap\ c\_2Erealax\_2Ereal\_add\ V0x)\ \\ & V1y)) = (ap\ (ap\ c\_2Erealax\_2Ereal\_add\ (ap\ c\_2Erealax\_2Ereal\_neg \\ & \quad V0x))\ (ap\ c\_2Erealax\_2Ereal\_neg\ V1y)))))) \end{aligned} \quad (51)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x) = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))) \quad (52)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V0x))) \quad (53)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. (\forall V2z \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V1y)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt V1y) V2z))) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt V0x) V2z)))))) \quad (54)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. (((p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V1y)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte V1y) V0x)))) \Leftrightarrow (V0x = V1y)))) \quad (55)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap c\_2Ereal\_2Eabs (ap c\_2Erealax\_2Ereal\_neg V0x)) = (ap c\_2Ereal\_2Eabs V0x))) \quad (56)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. (p (ap (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Eabs (ap (ap c\_2Ereal\_2Ereal\_sub (ap c\_2Ereal\_2Eabs V0x)) (ap c\_2Ereal\_2Eabs V1y)))) (ap c\_2Ereal\_2Eabs (ap (ap c\_2Ereal\_2Ereal\_sub V0x) V1y)))))) \quad (57)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Erealax\_2Ereal\_neg V0x)) V1y) = (ap c\_2Erealax\_2Ereal\_neg (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y)))))) \quad (58)$$

Assume the following.

$$(\forall V0y \in ty\_2Erealax\_2Ereal.(\forall V1x \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Erealax\_2Ereal\_lt V1x) V0y)) \Leftrightarrow (\neg (p (ap (ap c\_2Ereal\_2Ereal\_lte V0y) V1x)))))) \quad (59)$$



Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Erealax\_2Ereal\_neg V0x)) \\
& V1y)) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Erealax\_2Ereal\_neg V0x)) \\
& (ap c\_2Erealax\_2Ereal\_neg V1y))) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V1y) V0x))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap c\_2Erealax\_2Ereal\_neg \\
& (ap c\_2Erealax\_2Ereal\_neg V0x)) = V0x))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) (ap c\_2Erealax\_2Ereal\_neg \\
& V1y))) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap (ap c\_2Erealax\_2Ereal\_add \\
& V0x) V1y)) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))))))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y)) V2z) = (ap (ap c\_2Erealax\_2Ereal\_add \\
& (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V2z)) (ap (ap c\_2Erealax\_2Ereal\_mul \\
& V1y) V2z))))))
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((ap c\_2Ereal\_topology\_2EDist (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& ty\_2Erealax\_2Ereal) V0x) V1y)) = (ap c\_2Ereal\_2Eabs (ap (ap c\_2Ereal\_2Ereal\_sub \\
& V0x) V1y))))))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1s \in \\
& (2^{ty\_2Erealax\_2Ereal}).((p (ap (ap c\_2Ereal\_topology\_2Econtinuous\_on \\
& V0f) V1s)) \Leftrightarrow (\forall V2x \in ty\_2Erealax\_2Ereal.((p (ap (ap (c\_2Ebool\_2EIN \\
& ty\_2Erealax\_2Ereal) V2x) V1s)) \Rightarrow (\forall V3e \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) V3e)) \Rightarrow (\exists V4d \in ty\_2Erealax\_2Ereal.((p (ap \\
& (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \\
& V4d)) \wedge (\forall V5x\_27 \in ty\_2Erealax\_2Ereal.(((p (ap (ap (c\_2Ebool\_2EIN \\
& ty\_2Erealax\_2Ereal) V5x\_27) V1s)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& (ap c\_2Ereal\_2Eabs (ap (ap c\_2Ereal\_2Ereal\_sub V5x\_27) V2x))) \\
& V4d))) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Eabs (ap \\
& (ap c\_2Ereal\_2Ereal\_sub (ap V0f V5x\_27)) (ap V0f V2x)))) V3e))))))))) \\
& \hspace{15em} (66)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1s \in \\
& (2^{ty\_2Erealax\_2Ereal}).(((p (ap c\_2Ereal\_topology\_2Ecompact \\
& V1s)) \wedge ((\neg(V1s = (c\_2Epred\_set\_2EEMPTY ty\_2Erealax\_2Ereal))) \wedge \\
& (p (ap (ap c\_2Ereal\_topology\_2Econtinuous\_on V0f) V1s)))) \Rightarrow \\
& (\exists V2x \in ty\_2Erealax\_2Ereal.((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\
& V2x) V1s)) \wedge (\forall V3y \in ty\_2Erealax\_2Ereal.((p (ap (ap (c\_2Ebool\_2EIN \\
& ty\_2Erealax\_2Ereal) V3y) V1s)) \Rightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& (ap V0f V3y)) (ap V0f V2x))))))))) \\
& \hspace{15em} (67)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \hspace{15em} (68)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \hspace{15em} (69)$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \\
& \hspace{15em} (70)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \\
& \hspace{15em} (71)
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \hspace{15em} (72)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg( \\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{73}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{74}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{75}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (( \\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{76}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{77}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{78}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{79}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{80}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{81}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{82}$$

**Theorem 1**

$$\begin{aligned} & (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1a \in ty\_2Erealax\_2Ereal. \\ & ((p (ap c\_2Ereal\_topology\_2Ecompact V0s)) \wedge (\neg(V0s = (c\_2Epred\_set\_2EEMPTY \\ & \quad ty\_2Erealax\_2Ereal)))) \Rightarrow (\exists V2x \in ty\_2Erealax\_2Ereal.( \\ & \quad (p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V2x) V0s)) \wedge (\forall V3y \in \\ & \quad ty\_2Erealax\_2Ereal.((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\ & \quad V3y) V0s)) \Rightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_topology\_2EDist \\ & \quad (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\ & \quad V1a) V3y))) (ap c\_2Ereal\_topology\_2EDist (ap (ap (c\_2Epair\_2E\_2C \\ & \quad ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) V1a) V2x)))))))))) \end{aligned}$$