thm_2Ereal__topology_2EDIST__EQ (TMd93ghbXsZmNBwgn9cNC1mfshWLrfxpoPM)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \text{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \text{c\_2Ebool\_2EF}$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \tag{3}$$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{6}$$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 12 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 13 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Definition 14 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 15 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{7}$$

Definition 16 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 17 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 18 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ ($c_2Ebool_2E

Definition 19 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{8}$$

Definition 20 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 21 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{9}$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{10}$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{11}$$

Definition 22 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Definition 23 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{12}$$

Let $c\_2Ereal\_2Epow : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \tag{13}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{14}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{15}$$

Definition 24 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Ereal\_\_topology\_2EDist : \iota$ be given. Assume the following.

$$c\_2Ereal\_\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \tag{16}$$

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{17}$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{18}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{19}$$

Definition 25 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(t$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{20}$$

Definition 26 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Definition 27 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Assume the following.

$$True \tag{21}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Rightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Rightarrow True)\Leftrightarrow True)\wedge(((False\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge(((p\ V0t)\Rightarrow False)\Leftrightarrow(\neg(p\ V0t)))))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow True)) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y)\Leftrightarrow(V1y = V0x)))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow(p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{26}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{27}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow(((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))) \tag{28}$$

Assume the following.

$$(((ap\ \mathsf{c\_2Enum\_2ESUC}\ \mathsf{c\_2Earithmetic\_2EZERO}) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))\wedge((\forall V0n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)))\wedge(\forall V1n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1n)) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)))))) \tag{29}$$

Assume the following.

$$((\forall V0n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}$$
$$\mathsf{c\_2Enum\_2E0})\ V0n) = V0n))\wedge((\forall V1n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}$$
$$(\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ V1n)\ \mathsf{c\_2Enum\_2E0}) = V1n))\wedge((\forall V2n \in$$
$$\mathit{ty\_2Enum\_2Enum}.(\forall V3m \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}$$
$$(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V3m)) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ \mathsf{c\_2Enumeral\_2EiZ}\ (\mathit{ap}$$
$$(\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ V2n)\ V3m))))))\wedge((\forall V4n \in \mathit{ty\_2Enum\_2Enum}.$$
$$((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2A}\ \mathsf{c\_2Enum\_2E0})\ V4n) = \mathsf{c\_2Enum\_2E0}))\wedge$$
$$((\forall V5n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2A}$$
$$V5n)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V6n \in \mathit{ty\_2Enum\_2Enum}.$$
$$(\forall V7m \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2A}\ ($$
$$\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V6n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V7m)) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2A}$$
$$V6n)\ V7m)))))\wedge((\forall V8n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2D}$$
$$\mathsf{c\_2Enum\_2E0})\ V8n) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V9n \in \mathit{ty\_2Enum\_2Enum}.$$
$$((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2D}\ V9n)\ \mathsf{c\_2Enum\_2E0}) = V9n))\wedge((\forall V10n \in$$
$$\mathit{ty\_2Enum\_2Enum}.(\forall V11m \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2D}$$
$$(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V10n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V11m)) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2D}$$
$$V10n)\ V11m)))))\wedge((\forall V12n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2EEXP}$$
$$\mathsf{c\_2Enum\_2E0})\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V12n))) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V13n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}$$
$$(\mathit{ap}\ \mathit{c\_2Earithmetic\_2EEXP}\ \mathsf{c\_2Enum\_2E0})\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V13n))) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V14n \in$$
$$\mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2EEXP}\ V14n)\ \mathsf{c\_2Enum\_2E0}) =$$
$$(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\wedge$$
$$((\forall V15n \in \mathit{ty\_2Enum\_2Enum}.(\forall V16m \in \mathit{ty\_2Enum\_2Enum}.$$
$$((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2EEXP}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V15n))$$
$$(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V16m)) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2EEXP}\ V15n)\ V16m)))))\wedge(((\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC}$$
$$\mathsf{c\_2Enum\_2E0}) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$\mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V17n \in \mathit{ty\_2Enum\_2Enum}.($$
$$(\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V17n)) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC}\ V17n))))\wedge(((\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ \mathsf{c\_2Enum\_2E0}) =$$
$$\mathsf{c\_2Enum\_2E0})\wedge((\forall V18n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}$$
$$(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V18n)) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ V18n))))\wedge((\forall V19n \in \mathit{ty\_2Enum\_2Enum}.$$
$$(((\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V19n) = \mathsf{c\_2Enum\_2E0})\Leftrightarrow(V19n = \mathsf{c\_2Earithmetic\_2EZERO})))\wedge$$
$$((\forall V20n \in \mathit{ty\_2Enum\_2Enum}.((\mathsf{c\_2Enum\_2E0} = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V20n))\Leftrightarrow(V20n = \mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V21n \in \mathit{ty\_2Enum\_2Enum}.$$
$$(\forall V22m \in \mathit{ty\_2Enum\_2Enum}.(((\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V21n) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V22m))\Leftrightarrow(V21n = V22m))))\wedge$$
$$((\forall V23n \in \mathit{ty\_2Enum\_2Enum}.((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V23n)\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow \mathit{False}))\wedge((\forall V24n \in \mathit{ty\_2Enum\_2Enum}.$$
$$((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V24n)))\Leftrightarrow(p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})$$
$$V24n))))\wedge((\forall V25n \in \mathit{ty\_2Enum\_2Enum}.(\forall V26m \in \mathit{ty\_2Enum\_2Enum}.$$
$$((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V25n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V26m)))\Leftrightarrow(p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V25n)\ V26m)))))\wedge((\forall V27n \in \mathit{ty\_2Enum\_2Enum}.((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2E\_3E}$$
$$\mathsf{c\_2Enum\_2E0})\ V27n))\Leftrightarrow \mathit{False}))\wedge((\forall V28n \in \mathit{ty\_2Enum\_2Enum}.$$
$$((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V28n))\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow(p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})$$
$$V28n))))\wedge((\forall V29n \in \mathit{ty\_2Enum\_2Enum}.(\forall V30m \in \mathit{ty\_2Enum\_2Enum}.$$
$$((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V29n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V30m)))\Leftrightarrow(p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V30m)\ V29n)))))\wedge((\forall V31n \in \mathit{ty\_2Enum\_2Enum}.((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$\mathsf{c\_2Enum\_2E0})\ V31n))\Leftrightarrow \mathit{True}))\wedge((\forall V32n \in \mathit{ty\_2Enum\_2Enum}.$$
$$((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1x \in ty\_2Erealax\_2Ereal.(\forall V2y \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V1x)) \wedge ((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V2y)) \wedge ((ap\ (ap\ c\_2Ereal\_2Epow\ V1x)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n)) = (ap\ (ap\ c\_2Ereal\_2Epow\ V2y)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n))))) \Rightarrow (V1x = V2y))))) \quad (31)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ c\_2Ereal\_\_topology\_2EDist\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V0x)\ V1y)))))) \quad (32)$$

Theorem 1

$$(\forall V0w \in ty\_2Erealax\_2Ereal.(\forall V1x \in ty\_2Erealax\_2Ereal.(\forall V2y \in ty\_2Erealax\_2Ereal.(\forall V3z \in ty\_2Erealax\_2Ereal.(((ap\ c\_2Ereal\_\_topology\_2EDist\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V0w)\ V1x)) = (ap\ c\_2Ereal\_\_topology\_2EDist\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V2y)\ V3z))) \Leftrightarrow ((ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ c\_2Ereal\_\_topology\_2EDist\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V0w)\ V1x)))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))) = (ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ c\_2Ereal\_\_topology\_2EDist\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V2y)\ V3z)))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))))))))$$