

thm_2Ereal__topology_2EEQ__INTERVAL (TMX- eevaV7DmpumXAAYQP8Tvt1uXq84in8a6)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 4 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V0t \in 2.V0t)$.

Definition 5 We define `c_2Epred__set_2EEMPTY` to be $\lambda A. 27a : \iota. (\lambda V0x \in A. 27a. \text{c_2Ebool_2EF})$.

Let `ty_2Ehreal_2Ehreal` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Ehreal_2Ehreal} \tag{1}$$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A0 \ A1) \tag{2}$$

Let `ty_2Erealax_2Ereal` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Erealax_2Ereal} \tag{3}$$

Let `c_2Erealax_2Ereal__REP__CLASS` : ι be given. Assume the following.

$$\text{c_2Erealax_2Ereal_REP_CLASS} \in ((2^{(\text{ty_2Epair_2Eprod } \text{ty_2Ehreal_2Ehreal } \text{ty_2Ehreal_2Ehreal}) \text{ty_2Erealax}})) \tag{4}$$

Definition 6 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P \ x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge P \ x))$ of type $\iota \Rightarrow \iota$.

Definition 7 We define `c_2Erealax_2Ereal__REP` to be $\lambda V0a \in \text{ty_2Erealax_2Ereal}. (\text{ap } (\text{c_2Emin_2E_40 } (\text{ty_2Erealax_2Ereal_REP_CLASS } V0a)))$

Let `c_2Erealax_2Etrealt__lt` : ι be given. Assume the following.

$$\text{c_2Erealax_2Etrealt_lt} \in ((2^{(\text{ty_2Epair_2Eprod } \text{ty_2Ehreal_2Ehreal } \text{ty_2Ehreal_2Ehreal}) \text{ty_2Epair_2Eprod } \text{ty_2Ehreal}})) \tag{5}$$

Definition 8 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow P \Rightarrow Q)$ of type ι .

Definition 10 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 11 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 12 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).)(ap V1f V0x))$

Definition 13 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Ebool_2EIN A_27a) (V0s \subseteq V1t))$

Definition 14 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) (V0t1 \subseteq V1t2))))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} &\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod \\ &\quad A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (6)$$

Definition 15 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2E_2F_5C) (x \subseteq y))$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)}) \quad (7)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (8)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (10)$$

Definition 16 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (11)$$

Definition 17 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).)(ap V0P (ap (c_2Emin_2E_40) (P \subseteq V0P)))$

Definition 18 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Ebool_2E_3F) (s \subseteq ty_2Erealax_2Ereal))$

Definition 19 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E_2ET)$.

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}})$$
(12)

Definition 20 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EGSPEC\ A_27a\ A_27b)\ s\ t)$

Definition 21 We define $c_2Ereal_topology_2EClosed$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ c_2Ereal_topology_2EOPEN_interval\ s)$

Let $c_2Ereal_topology_2EOPEN_interval : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EOPEN_interval \in ((2^{ty_2Erealax_2Ereal})^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)})$$
(13)

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0)$$
(14)

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a)$$
(15)

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a})$$
(16)

Let $c_2Elist_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EHD\ A_27a \in (A_27a^{(ty_2Elist_2Elist\ A_27a)})$$
(17)

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$$
(18)

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$$
(19)

Definition 22 We define $c_2Ereal_topology_2ECLOSED_interval$ to be $\lambda V0l \in (ty_2Elist_2Elist\ (ty_2Epair_2ESND\ l\ l))$

Definition 23 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ t1\ t2))\ (\lambda V2t \in 2.t1\ t2))$

Assume the following.

$$True \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. ((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee \neg(p V0t))) \quad (23)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow \neg(p V0t))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2. (\neg(p V0t) \Rightarrow ((p V0t) \Rightarrow False))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2. (\neg(\neg(p V0t)) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee (p\ V1B) \wedge (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C)))))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V1B) \wedge (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A)))))) \quad (39)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (40)$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \Rightarrow (41)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V1y) V0x))) \Leftrightarrow (V0x = V1y)))) \quad (42)$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}).(((p (ap c_2Ereal_topology_2EClosed V0s)) \wedge (p (ap c_2Ereal_topology_2EOpen V0s))) \Leftrightarrow ((V0s = (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal)) \vee (V0s = (c_2Epred_set_2EUNIV ty_2Erealax_2Ereal)))))) \quad (43)$$

Assume the following.

$$(\forall V0a \in ty_2Erealax_2Ereal.(\forall V1b \in ty_2Erealax_2Ereal.(((\neg((ap c_2Ereal_topology_2ECLOSED_interval (ap (ap (c_2Elist_2ECONS ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)) (ap (ap (c_2Epair_2E2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0a) V1b)) (c_2Elist_2ENIL ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)))) = (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte V0a) V1b))) \wedge ((\neg((ap c_2Ereal_topology_2EOPEN_interval (ap (ap (c_2Epair_2E2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0a) V1b)) = (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal))) \Leftrightarrow (p (ap (ap c_2Erealax_2Ereal_lt V0a) V1b)))))) \quad (44)$$

Assume the following.

$$\begin{aligned}
& (\forall V0c \in ty_2Erealax_2Ereal. (\forall V1d \in ty_2Erealax_2Ereal. \\
& (\forall V2a \in ty_2Erealax_2Ereal. (\forall V3b \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) (ap \\
& c_2Ereal_topology_2ECLOSED_interval (ap (ap (c_2Elist_2ECONS \\
& (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)) \\
& (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V0c) V1d)) (c_2Elist_2ENIL (ty_2Epair_2Eprod ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal)))))) (ap c_2Ereal_topology_2ECLOSED_interval \\
& (ap (ap (c_2Elist_2ECONS (ty_2Epair_2Eprod ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal)) (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal) V2a) V3b)) (c_2Elist_2ENIL (ty_2Epair_2Eprod \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal)))))) \Leftrightarrow ((p (ap (ap c_2Ereal_2Ereal_lte \\
& V0c) V1d)) \Rightarrow ((p (ap (ap c_2Ereal_2Ereal_lte V2a) V0c)) \wedge (p (ap (\\
& ap c_2Ereal_2Ereal_lte V1d) V3b)))))) \wedge (((p (ap (ap (c_2Epred_set_2ESUBSET \\
& ty_2Erealax_2Ereal) (ap c_2Ereal_topology_2ECLOSED_interval \\
& (ap (ap (c_2Elist_2ECONS (ty_2Epair_2Eprod ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal)) (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal) V0c) V1d)) (c_2Elist_2ENIL (ty_2Epair_2Eprod \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal)))))) (ap c_2Ereal_topology_2EOPEN_interval \\
& (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V2a) V3b)))) \Leftrightarrow ((p (ap (ap c_2Ereal_2Ereal_lte V0c) V1d)) \Rightarrow ((p (\\
& ap (ap c_2Erealax_2Ereal_lt V2a) V0c)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt \\
& V1d) V3b)))))) \wedge (((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) \\
& (ap c_2Ereal_topology_2EOPEN_interval (ap (ap (c_2Epair_2E_2C \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0c) V1d)) (ap c_2Ereal_topology_2ECLOSED_interval \\
& (ap (ap (c_2Elist_2ECONS (ty_2Epair_2Eprod ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal)) (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal) V2a) V3b)) (c_2Elist_2ENIL (ty_2Epair_2Eprod \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal)))))) \Leftrightarrow ((p (ap (ap c_2Erealax_2Ereal_lt \\
& V0c) V1d)) \Rightarrow ((p (ap (ap c_2Ereal_2Ereal_lte V2a) V0c)) \wedge (p (ap (\\
& ap c_2Ereal_2Ereal_lte V1d) V3b)))))) \wedge ((p (ap (ap (c_2Epred_set_2ESUBSET \\
& ty_2Erealax_2Ereal) (ap c_2Ereal_topology_2EOPEN_interval \\
& (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V0c) V1d)) (ap c_2Ereal_topology_2EOPEN_interval (ap (ap (\\
& c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V2a) \\
& V3b)))) \Leftrightarrow ((p (ap (ap c_2Erealax_2Ereal_lt V0c) V1d)) \Rightarrow ((p (ap (\\
& ap c_2Ereal_2Ereal_lte V2a) V0c)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte \\
& V1d) V3b)))))))))
\end{aligned}$$

(45)

Assume the following.

$$(p (ap c.2Ereal_topology_2EOpen (ap c.2Ereal_topology_2EOPEN_interval (ap (ap (c.2Epair_2E_2C ty.2Erealax.2Ereal ty.2Erealax.2Ereal) V0a) V1b)))))) \quad (46)$$

Assume the following.

$$(p (ap c.2Ereal_topology_2EClosed (ap c.2Ereal_topology_2ECLOSED_interval (ap (ap (c.2Elist_2ECONS (ty.2Epair_2Eprod ty.2Erealax.2Ereal ty.2Erealax.2Ereal) (ap (ap (c.2Epair_2E_2C ty.2Erealax.2Ereal ty.2Erealax.2Ereal) V0a) V1b)) (c.2Elist_2ENIL (ty.2Epair_2Eprod ty.2Erealax.2Ereal ty.2Erealax.2Ereal))))))))) \quad (47)$$

Assume the following.

$$\begin{aligned} & ((\forall V0a \in ty.2Erealax.2Ereal. (\forall V1b \in ty.2Erealax.2Ereal. \\ & (\neg((ap c.2Ereal_topology_2ECLOSED_interval (ap (ap (c.2Elist_2ECONS \\ & (ty.2Epair_2Eprod ty.2Erealax.2Ereal ty.2Erealax.2Ereal) \\ & (ap (ap (c.2Epair_2E_2C ty.2Erealax.2Ereal ty.2Erealax.2Ereal) \\ & V0a) V1b)) (c.2Elist_2ENIL (ty.2Epair_2Eprod ty.2Erealax.2Ereal \\ & ty.2Erealax.2Ereal)))))) = (c.2Epred_set_2EUNIV ty.2Erealax.2Ereal)))))) \wedge \\ & (\forall V2a \in ty.2Erealax.2Ereal. (\forall V3b \in ty.2Erealax.2Ereal. \\ & (\neg((ap c.2Ereal_topology_2EOPEN_interval (ap (ap (c.2Epair_2E_2C \\ & ty.2Erealax.2Ereal ty.2Erealax.2Ereal) V2a) V3b)) = (c.2Epred_set_2EUNIV \\ & ty.2Erealax.2Ereal)))))) \quad (48) \end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (49)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (50)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (51)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (52)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (53)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{58}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{59}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{60}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{61}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{62}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{63}$$

