

thm_2Ereal_topology_2EEVENTUALLY_AT (TMK1PRGAWbnZzQWvJzBaDXaW9UrwUkZi8Fj)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 7 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E_2T)$.

Let $ty_2Ereal_topology_2Eenet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ereal_topology_2Eenet A0) \quad (1)$$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (2)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (3)$$

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2E_2F_5C$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (4)$$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (5)$$

Definition 10 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \quad (6)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (7)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (8)$$

Definition 11 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (9)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (10)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (11)$$

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x. x \in A) \text{ of type } \iota \Rightarrow \iota)$.

Definition 13 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal. (ap\ (c_2Emin_2E_40\ (ap\ P\ x)))$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (12)$$

Definition 14 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 15 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40\ (ap\ P\ x))))$

Definition 16 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Ereal_topology_2Emk_net : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ereal_topology_2Emk_net \\ A_27a \in ((ty_2Ereal_topology_2Enet\ A_27a)^{(2^{A_27a})^{A_27a}}) \end{aligned} \quad (13)$$

Definition 17 We define $c_2Ereal_topology_2Eat$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Ereal_topology_2Eat\ a))$

Let $c_2Ereal_topology_2Enetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ereal_topology_2Enetord\ A_27a \in ((2^{A_27a})^{A_27a})(ty_2Ereal_topology_2Enet\ A_27a) \quad (14)$$

Definition 18 We define $c_2Ereal_topology_2Ewithin$ to be $\lambda A_27a : \iota.\lambda V0net \in (ty_2Ereal_topology_2Ewithin\ A_27a\ net)$

Definition 19 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ t2))\ t1)$

Definition 20 We define $c_2Ereal_topology_2Etrivial_limit$ to be $\lambda A_27a : \iota.\lambda V0net \in (ty_2Ereal_topology_2Etrivial_limit\ A_27a\ net)$

Definition 21 We define $c_2Ereal_topology_2Eeventually$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1net \in (ty_2Ereal_topology_2Eeventually\ A_27a\ p\ net)$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (16)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (19)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg \\ (p\ V0t)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V0x)\ (c.2Epred_set.2EUNIV\ A.27a)))) \quad (21)$$

Assume the following.

$$(\forall V0x \in ty.2Erealax.2Ereal.((ap\ (ap\ (c.2Ereal_topology.2Ewithin\ ty.2Erealax.2Ereal)\ (ap\ c.2Ereal_topology.2Eat\ V0x))\ (c.2Epred_set.2EUNIV\ ty.2Erealax.2Ereal))) = (ap\ c.2Ereal_topology.2Eat\ V0x))) \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0s \in (2^{ty.2Erealax.2Ereal}).(\forall V1a \in ty.2Erealax.2Ereal. \\ & (\forall V2p \in (2^{ty.2Erealax.2Ereal}).((p\ (ap\ (ap\ (c.2Ereal_topology.2Eeventually\ ty.2Erealax.2Ereal)\ V2p)\ (ap\ (ap\ (c.2Ereal_topology.2Ewithin\ ty.2Erealax.2Ereal)\ (ap\ c.2Ereal_topology.2Eat\ V1a))\ V0s)))) \Leftrightarrow \\ & (\exists V3d \in ty.2Erealax.2Ereal.((p\ (ap\ (ap\ c.2Erealax.2Ereal_lt\ (ap\ c.2Ereal.2Ereal_of_num\ c.2Enum.2E0))\ V3d)) \wedge (\forall V4x \in \\ & ty.2Erealax.2Ereal.(((p\ (ap\ (ap\ (c.2Ebool.2EIN\ ty.2Erealax.2Ereal)\ V4x)\ V0s)) \wedge ((p\ (ap\ (ap\ c.2Erealax.2Ereal_lt\ (ap\ c.2Ereal.2Ereal_of_num\ c.2Enum.2E0))\ (ap\ c.2Ereal_topology.2EDist\ (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealax.2Ereal\ ty.2Erealax.2Ereal)\ V4x)\ V1a)))))) \wedge (p\ (ap\ (\\ & ap\ c.2Erealax.2Ereal_lt\ (ap\ c.2Ereal_topology.2EDist\ (ap\ (\\ & ap\ (c.2Epair.2E.2C\ ty.2Erealax.2Ereal\ ty.2Erealax.2Ereal)\ V4x)\ V1a)))\ V3d)))) \Rightarrow (p\ (ap\ V2p\ V4x)))))) \quad (23) \end{aligned}$$

Theorem 1

$$\begin{aligned} & (\forall V0a \in ty.2Erealax.2Ereal.(\forall V1p \in (2^{ty.2Erealax.2Ereal}). \\ & ((p\ (ap\ (ap\ (c.2Ereal_topology.2Eeventually\ ty.2Erealax.2Ereal)\ V1p)\ (ap\ c.2Ereal_topology.2Eat\ V0a))) \Leftrightarrow (\exists V2d \in ty.2Erealax.2Ereal. \\ & ((p\ (ap\ (ap\ c.2Erealax.2Ereal_lt\ (ap\ c.2Ereal.2Ereal_of_num\ c.2Enum.2E0))\ V2d)) \wedge (\forall V3x \in ty.2Erealax.2Ereal.(((p\ (\\ & ap\ (ap\ c.2Erealax.2Ereal_lt\ (ap\ c.2Ereal.2Ereal_of_num\ c.2Enum.2E0))\ (ap\ c.2Ereal_topology.2EDist\ (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealax.2Ereal\ ty.2Erealax.2Ereal)\ V3x)\ V0a)))))) \wedge (p\ (ap\ (ap\ c.2Erealax.2Ereal_lt\ (ap\ c.2Ereal_topology.2EDist\ (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealax.2Ereal\ ty.2Erealax.2Ereal)\ V3x)\ V0a)))\ V2d)))) \Rightarrow (p\ (ap\ V1p\ V3x)))))) \end{aligned}$$