

# thm\_2Ereal\_\_topology\_2EEVENTUALLY\_\_WITHIN\_\_INTERIOR (TML7RgS6GMpESF1Ao5hoyM7cbcWhYV1NYTp)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})_{ty\_2Erealax}) \tag{4}$$

**Definition 3** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$  then (the  $(\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ .

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})))$

**Definition 5** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E\_40 (ty$

Let  $c\_2Erealax\_2Etrealt\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealt\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})_{(ty\_2Epair\_2Eprod\ ty\_2Ehreal)}) \tag{5}$$

**Definition 6** We define  $c\_Erealax\_Ereal\_lt$  to be  $\lambda V0t1 \in ty\_Erealax\_Ereal.\lambda V1t2 \in ty\_Erealax\_Ereal$

**Definition 7** We define  $c\_Ebool\_EF$  to be  $(ap (c\_Ebool\_E21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 8** We define  $c\_Emin\_E3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 9** We define  $c\_Ebool\_E7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_Emin\_E3D\_3D\_3E V0t) c\_Ebool\_EF))$

**Definition 10** We define  $c\_Ereal\_Ereal\_lte$  to be  $\lambda V0x \in ty\_Erealax\_Ereal.\lambda V1y \in ty\_Erealax\_Ereal$

**Definition 11** We define  $c\_Ebool\_E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_E21 2) (\lambda V2t \in 2.V2t))))$

**Definition 12** We define  $c\_Ebool\_ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap (c\_Ebool\_E21 2) (\lambda V2t2 \in 2.V2t2))))))$

**Definition 13** We define  $c\_Ereal\_Emin$  to be  $\lambda V0x \in ty\_Erealax\_Ereal.\lambda V1y \in ty\_Erealax\_Ereal$

Let  $c\_Ereal\_topology\_Eball : \iota$  be given. Assume the following.

$$c\_Ereal\_topology\_Eball \in ((2^{ty\_Erealax\_Ereal})(ty\_Epair\_Eprod ty\_Erealax\_Ereal ty\_Erealax\_Ereal)) \quad (6)$$

**Definition 14** We define  $c\_Ebool\_EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 15** We define  $c\_Epred\_set\_ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_Ebool\_ECOND) (\lambda V2t \in 2.V2t)))$

Let  $c\_Epair\_EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_Epair\_EABS\_prod A\_27a A\_27b \in ((ty\_Epair\_Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (7)$$

**Definition 16** We define  $c\_Epair\_E2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_Ebool\_ECOND) (\lambda V2t \in 2.V2t)))$

Let  $c\_Ereal\_topology\_EDist : \iota$  be given. Assume the following.

$$c\_Ereal\_topology\_EDist \in (ty\_Erealax\_Ereal^{(ty\_Epair\_Eprod ty\_Erealax\_Ereal ty\_Erealax\_Ereal)}) \quad (8)$$

Let  $c\_Eenum\_EZERO\_REP : \iota$  be given. Assume the following.

$$c\_Eenum\_EZERO\_REP \in \omega \quad (9)$$

Let  $ty\_Eenum\_Eenum : \iota$  be given. Assume the following.

$$nonempty ty\_Eenum\_Eenum \quad (10)$$

Let  $c\_Eenum\_EABS\_num : \iota$  be given. Assume the following.

$$c\_Eenum\_EABS\_num \in (ty\_Eenum\_Eenum^{\omega}) \quad (11)$$

**Definition 17** We define  $c\_Eenum\_E0$  to be  $(ap c\_Eenum\_EABS\_num c\_Eenum\_EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (12)$$

**Definition 18** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 19** We define  $c\_2Ereal\_topology\_2EOpen$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}). (ap\ (c\_2Ebool\_2E\_2$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}}) \end{aligned} \quad (13)$$

**Definition 20** We define  $c\_2Ereal\_topology\_2Einterior$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}). (ap\ (c\_2Epred\_set\_2EGSPEC$

Let  $ty\_2Ereal\_topology\_2Eenet : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ereal\_topology\_2Eenet\ A0) \quad (14)$$

Let  $c\_2Ereal\_topology\_2Eenetord : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ereal\_topology\_2Eenetord\ A\_27a \in ((2^{A\_27a})^{A\_27a})^{(ty\_2Ereal\_topology\_2Eenet\ A\_27a)} \quad (15)$$

Let  $c\_2Ereal\_topology\_2Emk\_net : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ereal\_topology\_2Emk\_net \\ A\_27a \in ((ty\_2Ereal\_topology\_2Eenet\ A\_27a)^{(2^{A\_27a})^{A\_27a}}) \end{aligned} \quad (16)$$

**Definition 21** We define  $c\_2Ereal\_topology\_2Ewithin$  to be  $\lambda A\_27a : \iota. \lambda V0net \in (ty\_2Ereal\_topology\_2Emk\_net$

**Definition 22** We define  $c\_2Ereal\_topology\_2Eat$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal. (ap\ (c\_2Ereal\_topology\_2Ewithin$

**Definition 23** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2. ($

**Definition 24** We define  $c\_2Ereal\_topology\_2Etrivial\_limit$  to be  $\lambda A\_27a : \iota. \lambda V0net \in (ty\_2Ereal\_topology\_2Emk\_net$

**Definition 25** We define  $c\_2Ereal\_topology\_2Eeventually$  to be  $\lambda A\_27a : \iota. \lambda V0p \in (2^{A\_27a}). \lambda V1net \in (ty\_2Ereal\_topology\_2Emk\_net$

Assume the following.

$$True \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.((p \ V0t) \vee (\neg(p \ V0t)))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((p \ V0t) \Rightarrow False) \Rightarrow (\neg(p \ V0t)))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p \ V0t)) \Rightarrow ((p \ V0t) \Rightarrow False))) \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\ & (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\ & (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\ & (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (26)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (27)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ & p \ V0t)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\forall V1x \in A.27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p\ (ap\ V0P\ V2x)))))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge (((\neg(p\ V0A)) \vee (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \quad (33)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (34)$$

Assume the following.

$$2.(((p\ V0x) \Leftrightarrow (p\ V1x.27)) \wedge ((p\ V1x.27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y.27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x.27) \Rightarrow (p\ V3y.27)))) \quad (35)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(\forall V2z \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ V2z)\ (ap\ (ap\ c\_2Ereal\_2Emin\ V0x)\ V1y))) \Leftrightarrow ((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ V2z)\ V0x)) \wedge (p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ V2z)\ V1y)))))) \quad (36)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((ap\ c\_2Ereal\_topology\_2EDist\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V0x)\ V1y)) = (ap\ c\_2Ereal\_topology\_2EDist\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V1y)\ V0x)))) \quad (37)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2e \in ty\_2Erealax\_2Ereal. ((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\
V1y) (ap c\_2Ereal\_topology\_2Eball (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
ty\_2Erealax\_2Ereal) V0x) V2e)))) \Leftrightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt \\
(ap c\_2Ereal\_topology\_2EDist (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
ty\_2Erealax\_2Ereal) V0x) V1y))) V2e))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1s \in (2^{ty\_2Erealax\_2Ereal}). \\
& ((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V0x) (ap c\_2Ereal\_topology\_2Einterior \\
V1s))) \Leftrightarrow (\exists V2e \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
(ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V2e)) \wedge (p (ap (ap ( \\
c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) (ap c\_2Ereal\_topology\_2Eball \\
(ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
V0x) V2e))) V1s))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}). (\forall V1a \in ty\_2Erealax\_2Ereal. \\
& (\forall V2p \in (2^{ty\_2Erealax\_2Ereal}). ((p (ap (ap (c\_2Ereal\_topology\_2Eeventually \\
ty\_2Erealax\_2Ereal) V2p) (ap (ap (c\_2Ereal\_topology\_2Ewithin \\
ty\_2Erealax\_2Ereal) (ap c\_2Ereal\_topology\_2Eat V1a))) V0s))) \Leftrightarrow \\
& (\exists V3d \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
(ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V3d)) \wedge (\forall V4x \in \\
ty\_2Erealax\_2Ereal. (((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\
V4x) V0s)) \wedge ((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
c\_2Enum\_2E0)) (ap c\_2Ereal\_topology\_2EDist (ap (ap (c\_2Epair\_2E\_2C \\
ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) V4x) V1a)))) \wedge (p (ap ( \\
ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_topology\_2EDist (ap ( \\
ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) V4x) \\
V1a))) V3d)))) \Rightarrow (p (ap V2p V4x))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty\_2Erealax\_2Ereal. (\forall V1p \in (2^{ty\_2Erealax\_2Ereal}). \\
& ((p (ap (ap (c\_2Ereal\_topology\_2Eeventually ty\_2Erealax\_2Ereal) \\
V1p) (ap c\_2Ereal\_topology\_2Eat V0a))) \Leftrightarrow (\exists V2d \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
c\_2Enum\_2E0)) V2d)) \wedge (\forall V3x \in ty\_2Erealax\_2Ereal. (((p ( \\
ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \\
(ap c\_2Ereal\_topology\_2EDist (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
ty\_2Erealax\_2Ereal) V3x) V0a)))) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt \\
(ap c\_2Ereal\_topology\_2EDist (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
ty\_2Erealax\_2Ereal) V3x) V0a))) V2d)))) \Rightarrow (p (ap V1p V3x))))))
\end{aligned} \tag{41}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (44)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (45)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (51)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (52)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q))))) \quad (53)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p))))) \quad (54)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q))))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (56)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0p \in (2^{ty\_2Erealax\_2Ereal}). (\forall V1s \in (2^{ty\_2Erealax\_2Ereal}). \\ & (\forall V2x \in ty\_2Erealax\_2Ereal. ((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\ & V2x) (ap c\_2Ereal\_topology\_2Einterior V1s))) \Rightarrow ((p (ap (ap (c\_2Ereal\_topology\_2Eeventually \\ & ty\_2Erealax\_2Ereal) V0p) (ap (ap (c\_2Ereal\_topology\_2Ewithin \\ & ty\_2Erealax\_2Ereal) (ap c\_2Ereal\_topology\_2Eat V2x)) V1s))) \Leftrightarrow \\ & (p (ap (ap (c\_2Ereal\_topology\_2Eeventually ty\_2Erealax\_2Ereal) \\ & V0p) (ap c\_2Ereal\_topology\_2Eat V2x))))))) \end{aligned}$$