thm_2Ereal__topology_2EFINITE__INTERVAL (TMG1xJZ8Bgwx6DrovNxJGVgpi34eSnAVmb8)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^{A_27}$

Definition 5 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in 2$

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2EF

Definition 8 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 9 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 10 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{2}$$

Definition 11 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{3}$$

Definition 12 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 13 *We define* c_2Epred__set_2EDIFF *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2$

Definition 14 *We define* c_2Epred__set_2EDELETE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1x \in A\_27a.(ap\ (a$

Definition 15 *We define* c_2Epred__set_2EFINITE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap$ (c_2Ebool_2E_21 (2

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{4}$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{5}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{6}$$

Definition 16 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 17 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 (t

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{7}$$

Definition 18 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Definition 19 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{8}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{9}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{10}$$

Let $c\_2Elist\_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EHD\ A\_27a \in (A\_27a^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{11}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{12}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{13}$$

Definition 20 *We define* c_2Ereal__topology_2ECLOSED__interval *to be* $\lambda V0l \in (ty\_2Elist\_2Elist\ (ty\_2Epa$

Let $c\_2Ereal\_\_topology\_2EOPEN\_\_interval : \iota$ be given. Assume the following.

$$c\_2Ereal\_\_topology\_2EOPEN\_\_interval \in ((2^{ty\_2Erealax\_2Ereal})^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Ere} \tag{14}$$

Assume the following.

$$True \tag{15}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \tag{18}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{19}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{21}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B))))))) \tag{22}$$

Assume the following.

$$((\forall V0a \in ty\_2Erealax\_2Ereal.(\neg(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ ty\_2Erealax\_2Ereal)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ (\lambda V1x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ 2)\ V1x)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V0a)\ V1x))))))))\land((\forall V2a \in ty\_2Erealax\_2Ereal.(\neg(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ ty\_2Erealax\_2Ereal)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ (\lambda V3x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ 2)\ V3x)\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V2a)\ V3x))))))))\land((\forall V4b \in ty\_2Erealax\_2Ereal.(\neg(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ ty\_2Erealax\_2Ereal)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ (\lambda V5x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ 2)\ V5x)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V5x)\ V4b))))))))\land((\forall V6b \in ty\_2Erealax\_2Ereal.(\neg(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ ty\_2Erealax\_2Ereal)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ (\lambda V7x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ 2)\ V7x)\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V7x)\ V6b))))))))\land((\forall V8a \in ty\_2Erealax\_2Ereal.(\forall V9b \in ty\_2Erealax\_2Ereal.((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ ty\_2Erealax\_2Ereal)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ (\lambda V10x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ 2)\ V10x)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V8a)\ V10x))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V10x)\ V9b)))))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V9b)\ V8a)))))\land((\forall V11a \in ty\_2Erealax\_2Ereal.(\forall V12b \in ty\_2Erealax\_2Ereal.((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ ty\_2Erealax\_2Ereal)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ (\lambda V13x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ 2)\ V13x)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V11a)\ V13x))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V13x)\ V12b)))))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V12b)\ V11a)))))\land((\forall V14a \in ty\_2Erealax\_2Ereal.(\forall V15b \in ty\_2Erealax\_2Ereal.((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ ty\_2Erealax\_2Ereal)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ (\lambda V16x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ 2)\ V16x)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V14a)\ V16x))\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V16x)\ V15b)))))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V15b)\ V14a)))))\land(\forall V17a \in ty\_2Erealax\_2Ereal.(\forall V18b \in ty\_2Erealax\_2Ereal.((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ ty\_2Erealax\_2Ereal)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ (\lambda V19x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ 2)\ V19x)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V17a)\ V19x))\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V19x)\ V18b)))))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V18b)\ V17a)))))))))))) \quad (23)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\neg(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)))))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDIFF}\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \wedge (\neg(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t)))))))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(\forall V2s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V1y)\ V2s))) \Leftrightarrow ((V0x = V1y) \vee (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ V2s))))))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1x \in A\_27a.(\forall V2y \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDELETE}\ A\_27a)\ V0s)\ V2y))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1x)\ V0s)) \wedge (\neg(V1x = V2y))))))) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDELETE}\ A\_27a)\ V1s)\ V0x))) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V1s))))) \tag{29}$$

Assume the following.

$$(\forall V0a \in ty\_2Erealax\_2Ereal.(\forall V1b \in ty\_2Erealax\_2Ereal.(((ap\ c\_2Ereal\_\_topology\_2EOPEN\_\_interval\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V0a)\ V1b)) = (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ (\lambda V2x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ 2)\ V2x)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V0a)\ V2x))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V2x)\ V1b))))))\land((ap\ \mathsf{c\_2Ereal\_\_topology\_2ECLOSED\_\_interval}\ (ap\ (ap\ (c\_2Elist\_2ECONS\ (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V0a)\ V1b))\ (c\_2Elist\_2ENIL\ (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)))) = (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ (\lambda V3x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ 2)\ V3x)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0a)\ V3x))\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V3x)\ V1b))))))))) \tag{30}$$

Assume the following.

$$(\forall V0a \in ty\_2Erealax\_2Ereal.(\forall V1b \in ty\_2Erealax\_2Ereal.((ap\ c\_2Ereal\_\_topology\_2EOPEN\_\_interval\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V0a)\ V1b)) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDIFF}\ ty\_2Erealax\_2Ereal)\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2ECLOSED\_\_interval}\ (ap\ (ap\ (c\_2Elist\_2ECONS\ (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V0a)\ V1b))\ (c\_2Elist\_2ENIL\ (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)))))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ ty\_2Erealax\_2Ereal)\ V0a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ ty\_2Erealax\_2Ereal)\ V1b)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Erealax\_2Ereal))))))) \tag{31}$$

Theorem 1

$$((\forall V0a \in ty\_2Erealax\_2Ereal.(\forall V1b \in ty\_2Erealax\_2Ereal.((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ ty\_2Erealax\_2Ereal)\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2ECLOSED\_\_interval}\ (ap\ (ap\ (c\_2Elist\_2ECONS\ (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V0a)\ V1b))\ (c\_2Elist\_2ENIL\ (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal))))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1b)\ V0a)))))\land(\forall V2a \in ty\_2Erealax\_2Ereal.(\forall V3b \in ty\_2Erealax\_2Ereal.((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ ty\_2Erealax\_2Ereal)\ (ap\ c\_2Ereal\_\_topology\_2EOPEN\_\_interval\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V2a)\ V3b))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V3b)\ V2a))))))$$