

thm_2Ereal__topology_2EFRONTIER__CLOSED__INTERVAL (TMQ9BMbR7JDRCb1hLpBpyagTcy2HTw8gwy3)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Definition 8 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \tag{3}$$

Definition 11 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (4)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (5)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (6)$$

Definition 12 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (t$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (7)$$

Definition 13 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (8)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (10)$$

Definition 14 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (11)$$

Definition 15 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Ebool_2E_2$

Definition 16 We define $c_2Ereal_topology_2Elimit_point_of$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1s \in ($

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (12)$$

Definition 17 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21 2) (\lambda V2t \in$

Definition 18 We define $c_Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2E$

Definition 19 We define $c_Ereal_topology_2Eclosure$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (ap (c_2Epred_set_2E$

Definition 20 We define $c_Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2E$

Definition 21 We define $c_Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2E$

Definition 22 We define $c_Ereal_topology_2Einterior$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Epred_set_2E$

Definition 23 We define $c_Ereal_topology_2Efrontier$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (ap (c_2Epred_set_2E$

Definition 24 We define $c_Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_Ebool_2ET)$.

Definition 25 We define $c_Ereal_topology_2EClosed$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap c_Ereal_topo$

Let $c_Ereal_topology_2EOPEN_interval : \iota$ be given. Assume the following.

$$c_Ereal_topology_2EOPEN_interval \in ((2^{ty_2Erealax_2Ereal})^{(ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)}) \quad (13)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (14)$$

Let $c_Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (15)$$

Let $c_Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (16)$$

Let $c_Elist_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Elist_2EHD A_27a \in (A_27a)^{(ty_2Elist_2Elist A_27a)} \quad (17)$$

Let $c_Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Epair_2ESND A_27a A_27b \in (A_27b)^{(ty_2Epair_2Eprod A_27a A_27b)} \quad (18)$$

Definition 26 We define $c_Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Epair_2EFST A_27a A_27b \in (A_27a)^{(ty_2Epair_2Eprod A_27a A_27b)} \quad (19)$$

Definition 27 We define $c_Ereal_topology_2ECLOSED_interval$ to be $\lambda V0l \in (ty_2Elist_2Elist (ty_2Epa$

Assume the following.

$$True \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p V0t)))))) \end{aligned} \quad (23)$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}).((p (ap c_Ereal_topology_2EClosed V0s)) \Rightarrow ((ap c_Ereal_topology_2Eclosure V0s) = V0s))) \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty_2Erealax_2Ereal.(\forall V1b \in ty_2Erealax_2Ereal. \\ & (p (ap c_Ereal_topology_2EClosed (ap c_Ereal_topology_2ECLOSED_interval \\ & (ap (ap (c_2Elist_2ECONS (ty_2Epair_2Eprod ty_2Erealax_2Ereal \\ & ty_2Erealax_2Ereal)) (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\ & ty_2Erealax_2Ereal) V0a) V1b)) (c_2Elist_2ENIL (ty_2Epair_2Eprod \\ & ty_2Erealax_2Ereal ty_2Erealax_2Ereal)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty_2Erealax_2Ereal.(\forall V1b \in ty_2Erealax_2Ereal. \\ & ((ap c_Ereal_topology_2Einterior (ap c_Ereal_topology_2ECLOSED_interval \\ & (ap (ap (c_2Elist_2ECONS (ty_2Epair_2Eprod ty_2Erealax_2Ereal \\ & ty_2Erealax_2Ereal)) (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\ & ty_2Erealax_2Ereal) V0a) V1b)) (c_2Elist_2ENIL (ty_2Epair_2Eprod \\ & ty_2Erealax_2Ereal ty_2Erealax_2Ereal)))))) = (ap c_Ereal_topology_2EOPEN_interval \\ & (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\ & V0a) V1b)))) \end{aligned} \quad (26)$$

Theorem 1

$$\begin{aligned} & (\forall V0a \in ty_2Erealax_2Ereal. (\forall V1b \in ty_2Erealax_2Ereal. \\ & ((ap\ c_2Ereal_topology_2Efrontier\ (ap\ c_2Ereal_topology_2ECLOSED_interval \\ & \quad (ap\ (ap\ (c_2Elist_2ECONS\ (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal \\ & \quad \quad ty_2Erealax_2Ereal))\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal \\ & \quad \quad \quad ty_2Erealax_2Ereal)\ V0a)\ V1b))\ (c_2Elist_2ENIL\ (ty_2Epair_2Eprod \\ & \quad ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)))))) = (ap\ (ap\ (c_2Epred_set_2EDIFF \\ & \quad ty_2Erealax_2Ereal)\ (ap\ c_2Ereal_topology_2ECLOSED_interval \\ & \quad (ap\ (ap\ (c_2Elist_2ECONS\ (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal \\ & \quad \quad ty_2Erealax_2Ereal))\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal \\ & \quad \quad \quad ty_2Erealax_2Ereal)\ V0a)\ V1b))\ (c_2Elist_2ENIL\ (ty_2Epair_2Eprod \\ & \quad ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal))))))\ (ap\ c_2Ereal_topology_2EOPEN_interval \\ & \quad (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal) \\ & \quad \quad \quad V0a)\ V1b)))))) \end{aligned}$$