thm_2Ereal__topology_2EHAUSDIST__CLOSURE (TMEnrdQxxZBrM57fzpGaHn2Ry5A3aSjnJkw)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota{\Rightarrow}\iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A\wedge p$ *of type* $\iota{\Rightarrow}\iota$.

Definition 4 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Let $ty\_2Epair\_2Eprod : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}\forall A1.nonempty\ A1{\Rightarrow}nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (1)$$

Definition 5 *We define* c_2Epred__set_2EUNIV *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2ET})$.

Let $c\_2Epred\_\_set\_2EGSPEC : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \quad (2)$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (3)$$

Definition 6 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 7 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27a}$

Definition 8 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 9 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 10 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 11 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{4}$$

Definition 12 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Definition 13 *We define* c_2Epred__set_2EDIFF *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2$

Let $c\_2Ereal\_\_topology\_2EDist : \iota$ be given. Assume the following.

$$c\_2Ereal\_\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \tag{5}$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{6}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{7}$$

Definition 14 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(t$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehr} \tag{8}$$

Definition 15 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{9}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{10}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{11}$$

Definition 16 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{12}$$

Definition 17 *We define* c_2Ereal__topology_2EOpen *to be* $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap$ (c_2Ebool_2E_2

Definition 18 *We define* c_2Ereal__topology_2EClosed *to be* $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap$ c_2Ereal__topo

Definition 19 *We define* c_2Ereal__topology_2Econtinuous__on *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealc}$

Definition 20 *We define* c_2Ereal__topology_2Elimit__point__of *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1s \in ($

Definition 21 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 22 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c.$

Definition 23 *We define* c_2Ereal__topology_2Eclosure *to be* $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap\ (ap$ (c_2Epred

Let $c\_2Ereal\_\_topology\_2Ehausdist : \iota$ be given. Assume the following.

$$c\_2Ereal\_\_topology\_2Ehausdist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Er} \tag{13}$$

Definition 24 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 25 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Let $c\_2Ereal\_\_topology\_2Esetdist : \iota$ be given. Assume the following.

$$c\_2Ereal\_\_topology\_2Esetdist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal} \tag{14}$$

Definition 26 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Assume the following.

$$True \tag{15}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{18}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{19}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{23}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{24}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a))))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2a \in A\_27a.(\forall V3b \in A\_27b.(((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}).(\forall V1v \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1v)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V1v)\ \mathsf{c\_2Ebool\_2ET}) = (ap\ V0f\ V2x)))))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ A\_27a)))) \tag{30}$$

Assume the following.

$$(p\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2EClosed}\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ ty\_2Erealax\_2Ereal))) \tag{31}$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econtinuous\_\_on}\ (\lambda V1x \in ty\_2Erealax\_2Ereal.V1x))\ V0s))) \tag{32}$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V2t \in (2^{ty\_2Erealax\_2Ereal}).(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econtinuous\_\_on}\ V0f)\ V1s)) \wedge ((p\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2EClosed}\ V1s)) \wedge (p\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2EClosed}\ V2t)))) \Rightarrow (p\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2EClosed}\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ (\lambda V3x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ 2)\ V3x)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V3x)\ V1s))\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ (ap\ V0f\ V3x))\ V2t)))))))))) \tag{33}$$

Assume the following.

$$(\forall V0a \in ty\_2Erealax\_2Ereal.(p\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2EClosed}\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ (\lambda V1x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ 2)\ V1x)\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1x)\ V0a))))))) \tag{34}$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V2t \in (2^{ty\_2Erealax\_2Ereal}). (((p\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2EClosed}\ V2t))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econtinuous\_\_on}\ V0f)\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Eclosure}\ V1s))))\Rightarrow((\forall V3x \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V3x)\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Eclosure}\ V1s)))\Rightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ (ap\ V0f\ V3x))\ V2t))))\Leftrightarrow(\forall V4x \in ty\_2Erealax\_2Ereal. ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V4x)\ V1s))\Rightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ (ap\ V0f\ V4x))\ V2t))))))))) \tag{35}$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1t \in (2^{ty\_2Erealax\_2Ereal}). (p\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econtinuous\_\_on}\ (\lambda V2y \in ty\_2Erealax\_2Ereal. (ap\ c\_2Ereal\_\_topology\_2Esetdist\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal}))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ ty\_2Erealax\_2Ereal)\ V2y)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Erealax\_2Ereal)))\ V0s))))\ V1t)))) \tag{36}$$

Assume the following.

$$((\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1t \in (2^{ty\_2Erealax\_2Ereal}). ((ap\ c\_2Ereal\_\_topology\_2Esetdist\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal}))\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Eclosure}\ V0s))\ V1t)) = (ap\ c\_2Ereal\_\_topology\_2Esetdist\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal}))\ V0s)\ V1t)))))\wedge (\forall V2s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V3t \in (2^{ty\_2Erealax\_2Ereal}). ((ap\ c\_2Ereal\_\_topology\_2Esetdist\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal}))\ V2s)\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Eclosure}\ V3t))) = (ap\ c\_2Ereal\_\_topology\_2Esetdist\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal}))\ V2s)\ V3t)))))) \tag{37}$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1t \in (2^{ty_2Erealax_2Ereal}).
(\forall V2s_27 \in (2^{ty_2Erealax_2Ereal}).(\forall V3t_27 \in (2^{ty_2Erealax_2Ereal}).
((\forall V4b \in ty_2Erealax_2Ereal.(((\forall V5x \in ty_2Erealax_2Ereal.
((p\ (ap\ (ap\ (\mathsf{c_2Ebool_2EIN}\ ty_2Erealax_2Ereal)\ V5x)\ V0s))\Rightarrow(p\ (
ap\ (ap\ \mathsf{c_2Ereal_2Ereal__lte}\ (ap\ c_2Ereal__topology_2Esetdist
(ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))
(ap\ (ap\ (\mathsf{c_2Epred__set_2EINSERT}\ ty_2Erealax_2Ereal)\ V5x)\ (\mathsf{c_2Epred__set_2EEMPTY}
ty_2Erealax_2Ereal)))\ V1t)))\ V4b))))\wedge(\forall V6y \in ty_2Erealax_2Ereal.
((p\ (ap\ (ap\ (\mathsf{c_2Ebool_2EIN}\ ty_2Erealax_2Ereal)\ V6y)\ V1t))\Rightarrow(p\ (
ap\ (ap\ \mathsf{c_2Ereal_2Ereal__lte}\ (ap\ c_2Ereal__topology_2Esetdist
(ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))
(ap\ (ap\ (\mathsf{c_2Epred__set_2EINSERT}\ ty_2Erealax_2Ereal)\ V6y)\ (\mathsf{c_2Epred__set_2EEMPTY}
ty_2Erealax_2Ereal)))\ V0s)))\ V4b)))))\Leftrightarrow((\forall V7x \in ty_2Erealax_2Ereal.
((p\ (ap\ (ap\ (\mathsf{c_2Ebool_2EIN}\ ty_2Erealax_2Ereal)\ V7x)\ V2s_27))\Rightarrow
(p\ (ap\ (ap\ \mathsf{c_2Ereal_2Ereal__lte}\ (ap\ c_2Ereal__topology_2Esetdist
(ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))
(ap\ (ap\ (\mathsf{c_2Epred__set_2EINSERT}\ ty_2Erealax_2Ereal)\ V7x)\ (\mathsf{c_2Epred__set_2EEMPTY}
ty_2Erealax_2Ereal)))\ V3t_27)))\ V4b))))\wedge(\forall V8y \in ty_2Erealax_2Ereal.
((p\ (ap\ (ap\ (\mathsf{c_2Ebool_2EIN}\ ty_2Erealax_2Ereal)\ V8y)\ V3t_27))\Rightarrow
(p\ (ap\ (ap\ \mathsf{c_2Ereal_2Ereal__lte}\ (ap\ c_2Ereal__topology_2Esetdist
(ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))
(ap\ (ap\ (\mathsf{c_2Epred__set_2EINSERT}\ ty_2Erealax_2Ereal)\ V8y)\ (\mathsf{c_2Epred__set_2EEMPTY}
ty_2Erealax_2Ereal)))\ V2s_27)))\ V4b)))))))\Rightarrow((ap\ c_2Ereal__topology_2Ehausdist
(ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))
V0s)\ V1t)) = (ap\ c_2Ereal__topology_2Ehausdist\ (ap\ (ap\ (\mathsf{c_2Epair_2E_2C}
(2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))\ V2s_27)\ V3t_27))))))))) \tag{38}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{39}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{40}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{41}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{42}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{43}$$

Theorem 1

$$((\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1t \in (2^{ty\_2Erealax\_2Ereal}).$$
$$((ap\ c\_2Ereal\_\_topology\_2Ehausdist\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ($$
$$2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal}))\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Eclosure}$$
$$V0s))\ V1t)) = (ap\ c\_2Ereal\_\_topology\_2Ehausdist\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}$$
$$(2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal}))\ V0s)\ V1t)))))\wedge$$
$$(\forall V2s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V3t \in (2^{ty\_2Erealax\_2Ereal}).$$
$$((ap\ c\_2Ereal\_\_topology\_2Ehausdist\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ($$
$$2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal}))\ V2s)\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Eclosure}$$
$$V3t))) = (ap\ c\_2Ereal\_\_topology\_2Ehausdist\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}$$
$$(2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal}))\ V2s)\ V3t))))))$$