

thm_2Ereal__topology_2EHAUSDIST__NONTRIVIAL__ALT
(TMYYYDMvNwtCDEjbeYRPXtbfaRn-
qwtG23Gzi)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})\ ty_2Erealax_2Ereal) \tag{4}$$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))) (\lambda V1x \in 2.V1x)))$

Definition 5 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty_2Erealax_2Ereal)))$

Let $c_2Erealax_2Etrealt_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealt_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})\ (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) \tag{5}$$

Definition 6 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 7 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21) 2) (\lambda V0t \in 2.V0t)$.

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 10 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.V2t)$

Definition 12 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 13 We define c_2Ereal_2Emax to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$.

Definition 14 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (6)$$

Definition 15 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \end{aligned} \quad (7)$$

Definition 16 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal \\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \end{aligned} \quad (8)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \quad (9)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})} \quad (10)$$

Definition 17 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)$

Definition 18 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{11}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{12}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{13}$$

Definition 19 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \tag{14}$$

Definition 20 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECONJ$

Definition 21 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E40$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \tag{15}$$

Definition 22 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 23 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \tag{16}$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \tag{17}$$

Definition 24 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c)^{A_27a})$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal)^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)} \tag{18}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p\ V0t)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).(((p\ V0P) \wedge (\forall V2x \in A_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A_27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x))))))) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A_27a}).(((\forall V2x \in A_27a.((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in A_27a.(p\ (ap\ V1P\ V3x)) \vee (p\ V0Q))))))) \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2. \\ & (((((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27))))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\ & (((\neg(V0s = (c_2Epred_set_2EEMPTY\ ty_2Erealax_2Ereal)))) \wedge ((\\ & \neg(V1t = (c_2Epred_set_2EEMPTY\ ty_2Erealax_2Ereal)))) \wedge ((\exists V2b \in \\ & ty_2Erealax_2Ereal.(\forall V3x \in ty_2Erealax_2Ereal.((p\ (ap \\ & (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ V3x)\ V0s)) \Rightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\ & V3x)\ V2b)))))) \wedge ((\exists V4c \in ty_2Erealax_2Ereal.(\forall V5x \in \\ & ty_2Erealax_2Ereal.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal) \\ & V5x)\ V1t)) \Rightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ V5x)\ V4c)))))) \Rightarrow (\\ & (ap\ c_2Ereal_2Esup\ (ap\ (ap\ (c_2Epred_set_2EUNION\ ty_2Erealax_2Ereal) \\ & V0s)\ V1t)) = (ap\ (ap\ c_2Ereal_2Emax\ (ap\ c_2Ereal_2Esup\ V0s))\ (ap \\ & c_2Ereal_2Esup\ V1t)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). ((\exists V1x \in \\ & A.27a.(p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V1x)\ V0s))) \Leftrightarrow (\neg(V0s = (c.2Epred_set.2EEMPTY \\ & A.27a)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in \\ & A.27a. ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V0x)\ (ap\ (ap\ (c.2Epred_set.2EINSERT \\ & A.27a)\ V1y)\ (c.2Epred_set.2EEMPTY\ A.27a)))) \Leftrightarrow (V0x = V1y)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0s \in (2^{A.27a}). (\forall V1f \in (A.27b^{A.27a}). ((ap\ (ap\ (\\ & c.2Epred_set.2EIMAGE\ A.27a\ A.27b)\ V1f)\ V0s) = (c.2Epred_set.2EEMPTY \\ & A.27b)) \Leftrightarrow (V0s = (c.2Epred_set.2EEMPTY\ A.27a)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0P \in (2^{A.27a}). (\forall V1f \in (A.27a^{A.27b}). (\forall V2s \in \\ & (2^{A.27b}). ((\forall V3y \in A.27a. ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a) \\ & V3y)\ (ap\ (ap\ (c.2Epred_set.2EIMAGE\ A.27b\ A.27a)\ V1f)\ V2s)))) \Rightarrow (\\ & p\ (ap\ V0P\ V3y)))) \Leftrightarrow (\forall V4x \in A.27b. ((p\ (ap\ (ap\ (c.2Ebool.2EIN \\ & A.27b)\ V4x)\ V2s)) \Rightarrow (p\ (ap\ V0P\ (ap\ V1f\ V4x)))))))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty.2Erealax.2Ereal. (\forall V1y \in ty.2Erealax.2Ereal. \\ & (\forall V2z \in ty.2Erealax.2Ereal. (((p\ (ap\ (ap\ c.2Ereal.2Ereal_lte \\ & V0x)\ V1y)) \wedge (p\ (ap\ (ap\ c.2Ereal.2Ereal_lte\ V1y)\ V2z))) \Rightarrow (p\ (ap\ (\\ & ap\ c.2Ereal.2Ereal_lte\ V0x)\ V2z)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow \forall A.27e.nonempty \\
& A.27e \Rightarrow \forall A.27f.nonempty\ A.27f \Rightarrow \forall A.27g.nonempty\ A.27g \Rightarrow \\
& (\forall V0Q \in (2^{A.27b}).(\forall V1P \in (2^{A.27a}).(\forall V2f \in \\
& (A.27b^{A.27a}).(\forall V3z \in A.27b.((p\ (ap\ (ap\ (c.2Ebool.2EIN \\
& A.27b)\ V3z)\ (ap\ (c.2Epred_set.2EGSPEC\ A.27b\ A.27a)\ (\lambda V4x \in \\
& A.27a.(ap\ (ap\ (c.2Epair.2E.2C\ A.27b\ 2)\ (ap\ V2f\ V4x))\ (ap\ V1P\ V4x)))))) \Rightarrow \\
& (p\ (ap\ V0Q\ V3z))) \Leftrightarrow (\forall V5x \in A.27a.((p\ (ap\ V1P\ V5x)) \Rightarrow (p\ (ap\ V0Q \\
& (ap\ V2f\ V5x)))))) \wedge ((\forall V6P \in ((2^{A.27d})^{A.27c}).(\forall V7f \in \\
& ((A.27b^{A.27d})^{A.27c}).(\forall V8z \in A.27b.((p\ (ap\ (ap\ (c.2Ebool.2EIN \\
& A.27b)\ V8z)\ (ap\ (c.2Epred_set.2EGSPEC\ A.27b\ (ty.2Epair.2Eprod \\
& A.27c\ A.27d))\ (ap\ (c.2Epair.2EUNCURRY\ A.27c\ A.27d\ (ty.2Epair.2Eprod \\
& A.27b\ 2))\ (\lambda V9x \in A.27c.(\lambda V10y \in A.27d.(ap\ (ap\ (c.2Epair.2E.2C \\
& A.27b\ 2)\ (ap\ (ap\ V7f\ V9x)\ V10y))\ (ap\ (ap\ V6P\ V9x)\ V10y)))))) \Rightarrow (p \\
& (ap\ V0Q\ V8z))) \Leftrightarrow (\forall V11x \in A.27c.(\forall V12y \in A.27d.((p \\
& (ap\ (ap\ V6P\ V11x)\ V12y)) \Rightarrow (p\ (ap\ V0Q\ (ap\ (ap\ V7f\ V11x)\ V12y)))))) \wedge \\
& (\forall V13P \in (((2^{A.27g})^{A.27f})^{A.27e}).(\forall V14f \in (((A.27b^{A.27g})^{A.27f})^{A.27e}). \\
& (\forall V15z \in A.27b.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27b)\ V15z)\ (\\
& ap\ (c.2Epred_set.2EGSPEC\ A.27b\ (ty.2Epair.2Eprod\ A.27e\ (ty.2Epair.2Eprod \\
& A.27f\ A.27g)))\ (ap\ (c.2Epair.2EUNCURRY\ A.27e\ (ty.2Epair.2Eprod \\
& A.27f\ A.27g)\ (ty.2Epair.2Eprod\ A.27b\ 2))\ (\lambda V16w \in A.27e.(ap \\
& (c.2Epair.2EUNCURRY\ A.27f\ A.27g\ (ty.2Epair.2Eprod\ A.27b\ 2)) \\
& (\lambda V17x \in A.27f.(\lambda V18y \in A.27g.(ap\ (ap\ (c.2Epair.2E.2C\ A.27b \\
& 2)\ (ap\ (ap\ (ap\ V14f\ V16w)\ V17x)\ V18y))\ (ap\ (ap\ (ap\ V13P\ V16w)\ V17x \\
& V18y)))))) \Rightarrow (p\ (ap\ V0Q\ V15z))) \Leftrightarrow (\forall V19w \in A.27e.(\forall V20x \in \\
& A.27f.(\forall V21y \in A.27g.((p\ (ap\ (ap\ (ap\ V13P\ V19w)\ V20x)\ V21y)) \Rightarrow \\
& (p\ (ap\ V0Q\ (ap\ (ap\ (ap\ V14f\ V19w)\ V20x)\ V21y)))))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty.2Erealx.2Ereal.(\forall V1y \in ty.2Erealx.2Ereal. \\
& ((ap\ c.2Ereal_topology.2EDist\ (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealx.2Ereal \\
& ty.2Erealx.2Ereal)\ V0x)\ V1y)) = (ap\ c.2Ereal.2Eabs\ (ap\ (ap\ c.2Ereal.2Ereal_sub \\
& V0x)\ V1y))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty.2Erealx.2Ereal.(\forall V1y \in ty.2Erealx.2Ereal. \\
& ((ap\ c.2Ereal_topology.2EDist\ (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealx.2Ereal \\
& ty.2Erealx.2Ereal)\ V0x)\ V1y)) = (ap\ c.2Ereal_topology.2EDist \\
& (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealx.2Ereal\ ty.2Erealx.2Ereal) \\
& V1y)\ V0x))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& (((p (ap c_2Ereal_topology_2Ebunded_def V0s)) \wedge (p (ap c_2Ereal_topology_2Ebunded_def \\
& V1t)))) \Rightarrow (p (ap c_2Ereal_topology_2Ebunded_def (ap (c_2Epred_set_2EGSPEC \\
& ty_2Erealax_2Ereal (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)) \\
& (ap (c_2Epair_2EUNCURRY ty_2Erealax_2Ereal ty_2Erealax_2Ereal \\
& (ty_2Epair_2Eprod ty_2Erealax_2Ereal 2)) (\lambda V2x \in ty_2Erealax_2Ereal. \\
& (\lambda V3y \in ty_2Erealax_2Ereal.(ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& 2) (ap (ap c_2Ereal_2Ereal_sub V2x) V3y)) (ap (ap c_2Ebool_2E_2F_5C \\
& (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V2x) V0s)) (ap (ap (\\
& c_2Ebool_2EIN ty_2Erealax_2Ereal) V3y) V1t))))))))))))) \\
& \tag{43}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& (\forall V2x \in ty_2Erealax_2Ereal.(\forall V3y \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V2x) V0s)) \wedge (p \\
& (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V3y) V1t)))) \Rightarrow (p (ap \\
& (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_topology_2Esetdist (\\
& ap (ap (c_2Epair_2E_2C (2^{ty_2Erealax_2Ereal}) (2^{ty_2Erealax_2Ereal})) \\
& V0s) V1t))) (ap c_2Ereal_topology_2EDist (ap (ap (c_2Epair_2E_2C \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V2x) V3y))))))))) \\
& \tag{44}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& (((p (ap c_2Ereal_topology_2Ebunded_def V0s)) \wedge ((p (ap c_2Ereal_topology_2Ebunded_def \\
& V1t)) \wedge ((\neg(V0s = (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal))) \wedge \\
& (\neg(V1t = (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal)))))) \Rightarrow (\\
& (ap c_2Ereal_topology_2Ehausdist (ap (ap (c_2Epair_2E_2C (2^{ty_2Erealax_2Ereal}) \\
& (2^{ty_2Erealax_2Ereal})) V0s) V1t)) = (ap c_2Ereal_2Esup (ap (ap \\
& (c_2Epred_set_2EUNION ty_2Erealax_2Ereal) (ap (c_2Epred_set_2EGSPEC \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) (\lambda V2x \in ty_2Erealax_2Ereal. \\
& (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal 2) (ap c_2Ereal_topology_2Esetdist \\
& (ap (ap (c_2Epair_2E_2C (2^{ty_2Erealax_2Ereal}) (2^{ty_2Erealax_2Ereal})) \\
& (ap (ap (c_2Epred_set_2EINSERT ty_2Erealax_2Ereal) V2x) (c_2Epred_set_2EEMPTY \\
& ty_2Erealax_2Ereal))) V1t))) (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V2x) V0s)))))) (ap (c_2Epred_set_2EGSPEC ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal) (\lambda V3y \in ty_2Erealax_2Ereal.(ap (ap (c_2Epair_2E_2C \\
& ty_2Erealax_2Ereal 2) (ap c_2Ereal_topology_2Esetdist (ap \\
& (ap (c_2Epair_2E_2C (2^{ty_2Erealax_2Ereal}) (2^{ty_2Erealax_2Ereal})) \\
& (ap (ap (c_2Epred_set_2EINSERT ty_2Erealax_2Ereal) V3y) (c_2Epred_set_2EEMPTY \\
& ty_2Erealax_2Ereal))) V0s))) (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V3y) V1t))))))))) \\
& \tag{45}
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{46}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (47)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (48)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (49)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (51)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (52)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (53)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (54)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (57)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (58)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (59)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (60)$$

Theorem 1

$$\begin{aligned} & (\forall V0s \in (2^{ty_2Erealax_2Ereal}). (\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\ & (((p (ap c_2Ereal_topology_2Ebounded_def V0s)) \wedge ((p (ap c_2Ereal_topology_2Ebounded_def \\ & V1t)) \wedge ((\neg(V0s = (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal))) \wedge \\ & (\neg(V1t = (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal)))))) \Rightarrow (\\ & (ap c_2Ereal_topology_2Ehausdist (ap (ap (c_2Epair_2E_2C (2^{ty_2Erealax_2Ereal}) \\ & (2^{ty_2Erealax_2Ereal})) V0s) V1t)) = (ap (ap c_2Ereal_2Emax (ap \\ & c_2Ereal_2Esup (ap (c_2Epred_set_2EGSPEC ty_2Erealax_2Ereal \\ & ty_2Erealax_2Ereal) (\lambda V2x \in ty_2Erealax_2Ereal. (ap (ap (c_2Epair_2E_2C \\ & ty_2Erealax_2Ereal) 2) (ap c_2Ereal_topology_2Esetdist (ap \\ & (ap (c_2Epair_2E_2C (2^{ty_2Erealax_2Ereal}) (2^{ty_2Erealax_2Ereal})) \\ & (ap (ap (c_2Epred_set_2EINSERT ty_2Erealax_2Ereal) V2x) (c_2Epred_set_2EEMPTY \\ & ty_2Erealax_2Ereal))) V1t))) (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\ & V2x) V0s)))))) (ap c_2Ereal_2Esup (ap (c_2Epred_set_2EGSPEC \\ & ty_2Erealax_2Ereal ty_2Erealax_2Ereal) (\lambda V3y \in ty_2Erealax_2Ereal. \\ & (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal) 2) (ap c_2Ereal_topology_2Esetdist \\ & (ap (ap (c_2Epair_2E_2C (2^{ty_2Erealax_2Ereal}) (2^{ty_2Erealax_2Ereal})) \\ & (ap (ap (c_2Epred_set_2EINSERT ty_2Erealax_2Ereal) V3y) (c_2Epred_set_2EEMPTY \\ & ty_2Erealax_2Ereal))) V0s))) (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\ & V3y) V1t)))))))))) \end{aligned}$$