

thm_2Ereal__topology_2EHAUSDIST__POS__LE
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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ecombin_2E_2S$ to be $\lambda A.\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 5 We define $c_2Ecombin_2E_2C$ to be $\lambda A.\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a})).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (P)))$

Definition 7 We define $c_2Ecombin_2E_2Eo$ to be $\lambda A.\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in (A_27b^{A_27c})).\lambda V1g \in (A_27a^{A_27c})$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (2)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (3)$$

Definition 8 We define $c_2Epair_2EUNCURRY$ to be $\lambda A.\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{4}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{5}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{6}$$

Definition 9 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{7}$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \tag{8}$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{9}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \tag{10}$$

Definition 10 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ ($

Let $c_2Erealax_2Etrealt_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealt_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \tag{11}$$

Definition 11 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 12 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 13 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2$

Definition 14 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ ($

Definition 15 We define c_2Ereal_2Esup to be $\lambda V0P \in (2^{ty_2Erealax_2Ereal}).(ap\ (c_2Emin_2E_40\ ty_2Ereal$

Definition 16 We define $c_2Ebool_2E_7E$ to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 17 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E$

Definition 18 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Er$

Definition 19 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 20 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 21 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (12)$$

Definition 22 We define c_2Epair_2E2C to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (13)$$

Definition 23 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap\ (c_2$

Let $c_2Ereal_topology_2Esetdist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2Esetdist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal})$$

Definition 24 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2$

Definition 25 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Let $c_2Ereal_topology_2Ehausdist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2Ehausdist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Er} \quad (15)$$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee \neg(p\ V0t))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p\ V0t)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\ & A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ & V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF) \\ & V0t1)\ V1t2) = V1t2)))) \end{aligned} \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg(\forall V1x \in A_27a. (p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\exists V2x \in A_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \wedge (\exists V3x \in A_27a. (p\ (ap\ V1Q\ V3x))))))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \wedge (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C))))))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V1B) \wedge (p\ V2C) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A))))))) \quad (31)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (32)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27)\ V5y_27)))))) \quad (34)$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}). (\forall V1a \in ty_2Erealax_2Ereal. (\forall V2b \in ty_2Erealax_2Ereal. (\forall V3y \in ty_2Erealax_2Ereal. (((p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ V3y)\ V0s)) \wedge ((p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ V1a)\ V3y)) \wedge (\forall V4x \in ty_2Erealax_2Ereal. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ V4x)\ V0s)) \Rightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ V4x)\ V2b)))))) \Rightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ V1a)\ (ap\ c_2Ereal_2Esup\ V0s)))))) \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t)))))) \quad (36)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg (p (ap (ap (c_2Ebool_2EIN\ A_27a)\ V0x)\ (c_2Epred_set_2EEMPTY\ A_27a)))))) \quad (37)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1s \in \\ (2^{A_27a}). (\forall V2t \in (2^{A_27a}). ((\forall V3x \in A_27a. ((p (\\ ap (ap (c_2Ebool_2EIN\ A_27a)\ V3x)\ (ap (ap (c_2Epred_set_2EUNION \\ A_27a)\ V1s)\ V2t)))) \Rightarrow (p (ap\ V0P\ V3x)))) \Leftrightarrow ((\forall V4x \in A_27a. ((p \\ (ap (ap (c_2Ebool_2EIN\ A_27a)\ V4x)\ V1s)) \Rightarrow (p (ap\ V0P\ V4x)))) \wedge (\forall V5x \in \\ A_27a. ((p (ap (ap (c_2Ebool_2EIN\ A_27a)\ V5x)\ V2t)) \Rightarrow (p (ap\ V0P\ V5x)))))))))) \end{aligned} \quad (38)$$

Assume the following.

$$(\forall V0x \in ty_2Erealx_2Ereal. (p (ap (ap\ c_2Ereal_2Ereal_lte\ V0x)\ V0x))) \quad (39)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow \forall A_27e.nonempty \\ A_27e \Rightarrow \forall A_27f.nonempty\ A_27f \Rightarrow \forall A_27g.nonempty\ A_27g \Rightarrow \\ (\forall V0Q \in (2^{A_27b}). ((\forall V1P \in (2^{A_27a}). (\forall V2f \in \\ (A_27b^{A_27a}). ((\forall V3z \in A_27b. ((p (ap (ap (c_2Ebool_2EIN \\ A_27b)\ V3z)\ (ap (c_2Epred_set_2EGSPEC\ A_27b\ A_27a)\ (\lambda V4x \in \\ A_27a. (ap (ap (c_2Epair_2E_2C\ A_27b\ 2)\ (ap\ V2f\ V4x)\ (ap\ V1P\ V4x)))))) \Rightarrow \\ (p (ap\ V0Q\ V3z)))) \Leftrightarrow (\forall V5x \in A_27a. ((p (ap\ V1P\ V5x)) \Rightarrow (p (ap\ V0Q \\ (ap\ V2f\ V5x)))))) \wedge ((\forall V6P \in ((2^{A_27d})^{A_27e}). (\forall V7f \in \\ ((A_27b^{A_27d})^{A_27e}). ((\forall V8z \in A_27b. ((p (ap (ap (c_2Ebool_2EIN \\ A_27b)\ V8z)\ (ap (c_2Epred_set_2EGSPEC\ A_27b\ (ty_2Epair_2Eprod \\ A_27c\ A_27d))\ (ap (c_2Epair_2EUNCURRY\ A_27c\ A_27d)\ (ty_2Epair_2Eprod \\ A_27b\ 2))\ (\lambda V9x \in A_27c. (\lambda V10y \in A_27d. (ap (ap (c_2Epair_2E_2C \\ A_27b\ 2)\ (ap (ap\ V7f\ V9x)\ V10y)\ (ap (ap\ V6P\ V9x)\ V10y)))))) \Rightarrow (p \\ (ap\ V0Q\ V8z)))) \Leftrightarrow (\forall V11x \in A_27c. (\forall V12y \in A_27d. ((p \\ (ap (ap\ V6P\ V11x)\ V12y)) \Rightarrow (p (ap\ V0Q\ (ap (ap\ V7f\ V11x)\ V12y)))))) \wedge \\ (\forall V13P \in (((2^{A_27g})^{A_27f})^{A_27e}). (\forall V14f \in (((A_27b^{A_27g})^{A_27f})^{A_27e}). \\ ((\forall V15z \in A_27b. ((p (ap (ap (c_2Ebool_2EIN\ A_27b)\ V15z)\ (\\ ap (c_2Epred_set_2EGSPEC\ A_27b\ (ty_2Epair_2Eprod\ A_27e\ (ty_2Epair_2Eprod \\ A_27f\ A_27g)))\ (ap (c_2Epair_2EUNCURRY\ A_27e\ (ty_2Epair_2Eprod \\ A_27f\ A_27g)\ (ty_2Epair_2Eprod\ A_27b\ 2))\ (\lambda V16w \in A_27e. (ap \\ (c_2Epair_2EUNCURRY\ A_27f\ A_27g)\ (ty_2Epair_2Eprod\ A_27b\ 2)) \\ (\lambda V17x \in A_27f. (\lambda V18y \in A_27g. (ap (ap (c_2Epair_2E_2C\ A_27b \\ 2)\ (ap (ap (ap\ V14f\ V16w)\ V17x)\ V18y)\ (ap (ap (ap\ V13P\ V16w)\ V17x) \\ V18y)))))) \Rightarrow (p (ap\ V0Q\ V15z)))) \Leftrightarrow (\forall V19w \in A_27e. (\forall V20x \in \\ A_27f. (\forall V21y \in A_27g. ((p (ap (ap (ap\ V13P\ V19w)\ V20x)\ V21y)) \Rightarrow \\ (p (ap\ V0Q\ (ap (ap (ap\ V14f\ V19w)\ V20x)\ V21y))))))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty-2Erealax-2Ereal}), (\forall V1t \in (2^{ty-2Erealax-2Ereal}), \\
& (p (ap (ap (ap c-2Ereal-2Ereal_lte (ap c-2Ereal-2Ereal_of_num \\
& c-2Enum-2E0)) (ap c-2Ereal_topology-2Esetdist (ap (ap (c-2Epair-2E-2C \\
& (2^{ty-2Erealax-2Ereal}) (2^{ty-2Erealax-2Ereal})) V0s) V1t))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& ((ap\ c_2Ereal_topology_2Ehausdist\ (ap\ (ap\ (c_2Epair_2E_2C\ (\\
& \quad 2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))\ V0s)\ V1t))) = (\\
& \quad ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ ty_2Erealax_2Ereal)\ (ap\ (ap\ c_2Ebool_2E_2F_5C \\
& \quad \quad (ap\ c_2Ebool_2E_7E\ (ap\ (ap\ (c_2Emin_2E_3D\ (2^{ty_2Erealax_2Ereal})) \\
& (ap\ (ap\ (c_2Epred_set_2EUNION\ ty_2Erealax_2Ereal)\ (ap\ (c_2Epred_set_2EGSPEC \\
& \quad ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ (\lambda V2x \in ty_2Erealax_2Ereal. \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ 2)\ (ap\ c_2Ereal_topology_2Esetdist \\
& \quad \quad (ap\ (ap\ (c_2Epair_2E_2C\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal})) \\
& (ap\ (ap\ (c_2Epred_set_2EINSERT\ ty_2Erealax_2Ereal)\ V2x)\ (c_2Epred_set_2EEMPTY \\
& \quad ty_2Erealax_2Ereal))))\ V1t)))\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal \\
& \quad \quad V2x)\ V0s))))\ (ap\ (c_2Epred_set_2EGSPEC\ ty_2Erealax_2Ereal \\
& \quad ty_2Erealax_2Ereal)\ (\lambda V3y \in ty_2Erealax_2Ereal.(ap\ (ap\ (c_2Epair_2E_2C \\
& \quad ty_2Erealax_2Ereal\ 2)\ (ap\ c_2Ereal_topology_2Esetdist\ (ap \\
& \quad \quad (ap\ (c_2Epair_2E_2C\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal})) \\
& (ap\ (ap\ (c_2Epred_set_2EINSERT\ ty_2Erealax_2Ereal)\ V3y)\ (c_2Epred_set_2EEMPTY \\
& \quad ty_2Erealax_2Ereal))))\ V0s)))\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal \\
& \quad \quad V3y)\ V1t))))\ (c_2Epred_set_2EEMPTY\ ty_2Erealax_2Ereal)))) \\
& \quad (ap\ (c_2Ebool_2E_3F\ ty_2Erealax_2Ereal)\ (\lambda V4b \in ty_2Erealax_2Ereal. \\
& \quad (ap\ (c_2Ebool_2E_21\ ty_2Erealax_2Ereal)\ (\lambda V5d \in ty_2Erealax_2Ereal. \\
& \quad (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal \\
& \quad \quad V5d)\ (ap\ (ap\ (c_2Epred_set_2EUNION\ ty_2Erealax_2Ereal)\ (ap\ (\\
& \quad \quad \quad c_2Epred_set_2EGSPEC\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal \\
& \quad \quad \quad (\lambda V6x \in ty_2Erealax_2Ereal.(ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal \\
& \quad \quad \quad \quad 2)\ (ap\ c_2Ereal_topology_2Esetdist\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad \quad \quad \quad \quad 2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))\ (ap\ (ap\ (c_2Epred_set_2EINSERT \\
& \quad \quad \quad ty_2Erealax_2Ereal)\ V6x)\ (c_2Epred_set_2EEMPTY\ ty_2Erealax_2Ereal))) \\
& \quad \quad \quad V1t)))\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ V6x)\ V0s)))) \\
& \quad (ap\ (c_2Epred_set_2EGSPEC\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal \\
& \quad (\lambda V7y \in ty_2Erealax_2Ereal.(ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal \\
& \quad \quad \quad 2)\ (ap\ c_2Ereal_topology_2Esetdist\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad \quad \quad \quad 2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))\ (ap\ (ap\ (c_2Epred_set_2EINSERT \\
& \quad \quad \quad ty_2Erealax_2Ereal)\ V7y)\ (c_2Epred_set_2EEMPTY\ ty_2Erealax_2Ereal))) \\
& \quad \quad \quad V0s)))\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ V7y)\ V1t))))\ (\\
& \quad \quad (ap\ (ap\ c_2Ereal_2Ereal_lte\ V5d)\ V4b))))\ (ap\ c_2Ereal_2Esup \\
& (ap\ (ap\ (c_2Epred_set_2EUNION\ ty_2Erealax_2Ereal)\ (ap\ (c_2Epred_set_2EGSPEC \\
& \quad ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ (\lambda V8x \in ty_2Erealax_2Ereal. \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ 2)\ (ap\ c_2Ereal_topology_2Esetdist \\
& \quad \quad (ap\ (ap\ (c_2Epair_2E_2C\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal})) \\
& (ap\ (ap\ (c_2Epred_set_2EINSERT\ ty_2Erealax_2Ereal)\ V8x)\ (c_2Epred_set_2EEMPTY \\
& \quad ty_2Erealax_2Ereal))))\ V1t)))\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal \\
& \quad \quad V8x)\ V0s))))\ (ap\ (c_2Epred_set_2EGSPEC\ ty_2Erealax_2Ereal \\
& \quad ty_2Erealax_2Ereal)\ (\lambda V9y \in ty_2Erealax_2Ereal.(ap\ (ap\ (c_2Epair_2E_2C \\
& \quad ty_2Erealax_2Ereal\ 2)\ (ap\ c_2Ereal_topology_2Esetdist\ (ap \\
& \quad \quad (ap\ (c_2Epair_2E_2C\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal})) \\
& (ap\ (ap\ (c_2Epred_set_2EINSERT\ ty_2Erealax_2Ereal)\ V9y)\ (c_2Epred_set_2EEMPTY \\
& \quad ty_2Erealax_2Ereal))))\ V0s)))\ (\&ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal \\
& \quad \quad V9y)\ V1t))))\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)))) \\
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (44)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (45)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (46)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p))))))))))))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (51)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (52)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (53)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (54)$$

Theorem 1

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}). (\forall V1t \in (2^{ty_2Erealax_2Ereal}). (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) (ap c_2Ereal_topology_2Ehausdist (ap (ap (c_2Epair_2E_2C (2^{ty_2Erealax_2Ereal}) (2^{ty_2Erealax_2Ereal})) V0s) V1t))))))$$