

thm_2Ereal__topology_2EHAUSDIST__SINGS
(TMXL62wRgRqNcG7k5ngtUvtN9EEQp9DqmFM)

October 26, 2020

Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty ty_2Erealax_2Ereal \quad (2)$$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)}) \quad (3)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (4)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (5)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (6)$$

Definition 4 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (7)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (8)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{ty_2Erealax}) \quad (9)$$

Definition 5 We define c_2Ebool_2E21 to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ (ap\ (c_2Emin_2E3D\ (2^{A_27a})))$

Definition 6 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal. (ap\ (c_2Emin_2E40\ (ty_2Erealax_2Ereal\ V0a)))$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (10)$$

Definition 7 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal. (c_2Erealax_2Ereal_lt\ V0T1\ V1T2)$

Definition 8 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2. (c_2Ebool_2E2F_5C\ V0t1\ V1t2\ V2t))))$

Definition 10 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E40\ (ty_2Erealax_2Ereal\ V0P))))$

Definition 11 We define c_2Ereal_2Esup to be $\lambda V0P \in (2^{ty_2Erealax_2Ereal}). (ap\ (c_2Emin_2E40\ ty_2Erealax_2Ereal\ V0P))$

Definition 12 We define c_2Ebool_2E2F to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2. V0t))$.

Definition 13 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E2F\ V0t))$

Definition 14 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal. \lambda V1y \in ty_2Erealax_2Ereal. (c_2Ereal_2Ereal_lte\ V0x\ V1y)$

Definition 15 We define c_2Ebool_2E2IN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Definition 16 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2E2F)$.

Definition 17 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2. (c_2Ebool_2E5C_2F\ V0t1\ V1t2\ V2t))))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (11)$$

Definition 18 We define `c_2Epair_2E_2C` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \quad (12)$$

Definition 19 We define `c_2Epred_set_2EINSERT` to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2Ereal_topology_2Esetdist : \iota)$ be given. Assume the following.

$$c_2Ereal_topology_2Esetdist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))}) \quad (13)$$

Definition 20 We define `c_2Epred_set_2EUNION` to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Ereal_topology_2Ehausdist : \iota)$ be given. Assume the following.

$$c_2Ereal_topology_2Ehausdist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))}) \quad (14)$$

Definition 21 We define `c_2Ebool_2ECOND` to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($ Assume the following.

$$True \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))) \quad (19) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee \\ & (p\ V0t)) \Leftrightarrow (p\ V0t))))) \quad (20) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ & A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in \\ & 2.(((\forall V2x \in A_27a.(p (ap V0P V2x))) \wedge (p V1Q)) \Leftrightarrow (\forall V3x \in \\ & A_27a.((p (ap V0P V3x)) \wedge (p V1Q)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \forall V0b \in 2.(\forall V1f \in (A_27b^{A_27a}).(\forall V2g \in (A_27b^{A_27a}). \\ & (\forall V3x \in A_27a.((ap (ap (ap (ap (c_2Ebool_2ECOND (A_27b^{A_27a})) \\ & V0b) V1f) V2g) V3x) = (ap (ap (ap (c_2Ebool_2ECOND A_27b) V0b) (ap \\ & V1f V3x)) (ap V2g V3x)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in (A_27b^{A_27a}). (\forall V1b \in 2. (\forall V2x \in A_27a. \\ & \quad (\forall V3y \in A_27a. ((ap\ V0f\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\ & \quad V1b)\ V2x)\ V3y)) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27b)\ V1b)\ (ap\ V0f \\ & \quad V2x))\ (ap\ V0f\ V3y))))))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ & \quad 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))))) \Rightarrow \\ & \quad (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & \quad (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ & \quad (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & \quad ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\ & \quad V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27 \\ & \quad V5y_27))))))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in \\ & \quad A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (\\ & \quad ap\ V0P\ V1a)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in (2^{A_27a}). (\forall V1v \in \\ & \quad A_27a. ((\forall V2x \in A_27a. ((V2x = V1v) \Rightarrow (p\ (ap\ V0f\ V2x)))) \Leftrightarrow (p\ (\\ & \quad ap\ V0f\ V1v)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty_2Erealax_2Ereal. ((ap\ c_2Ereal_2Esup\ (ap\ (ap \\ & \quad (c_2Epred_set_2EINSERT\ ty_2Erealax_2Ereal)\ V0a)\ (c_2Epred_set_2EEMPTY \\ & \quad ty_2Erealax_2Ereal)) = V0a)) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\ & \quad A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\ & \quad (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN \\ & A_27a) V2x) V0s) \Leftrightarrow (p (ap (ap (c_2Ebool_2EIN A_27a) V2x) V1t))))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}). (\forall V1v \in \\ & A_27a. ((p (ap (ap (c_2Ebool_2EIN A_27a) V1v) (ap (c_2Epred_set_2EGSPEC \\ & A_27a\ A_27b) V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap (ap (c_2Epair_2E_2C \\ & A_27a\ 2) V1v) c_2Ebool_2ET) = (ap V0f V2x)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg (p (ap (ap (c_2Ebool_2EIN A_27a) V0x) (c_2Epred_set_2EEMPTY A_27a)))))) \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). ((ap (ap (c_2Epred_set_2EUNION A_27a) V0s) V0s) = V0s)) \quad (40)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ & A_27a. (\forall V2s \in (2^{A_27a}). ((p (ap (ap (c_2Ebool_2EIN A_27a) \\ & V0x) (ap (ap (c_2Epred_set_2EINSERT A_27a) V1y) V2s))) \Leftrightarrow ((V0x = \\ & V1y) \vee (p (ap (ap (c_2Ebool_2EIN A_27a) V0x) V2s))))))) \end{aligned} \quad (41)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1s \in (2^{A_27a}). (\neg ((ap (ap (c_2Epred_set_2EINSERT A_27a) V0x) V1s) = (c_2Epred_set_2EEMPTY A_27a)))))) \quad (42)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ & A_27a. ((p (ap (ap (c_2Ebool_2EIN A_27a) V0x) (ap (ap (c_2Epred_set_2EINSERT \\ & A_27a) V1y) (c_2Epred_set_2EEMPTY A_27a)))) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (43)$$

Assume the following.

$$(\forall V0x \in ty_2Erealx_2Ereal. (p (ap (ap c_2Ereal_2Ereal_lte V0x) V0x))) \quad (44)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap\ c_2Ereal_topology_2EDist\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal)\ V0x)\ V1y)) = (ap\ c_2Ereal_topology_2EDist \\
& (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal) \\
& V1y)\ V0x))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap\ c_2Ereal_topology_2Esetdist\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{ty_2Erealax_2Ereal}) \\
& (2^{ty_2Erealax_2Ereal}))\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ ty_2Erealax_2Ereal) \\
& V0x)\ (c_2Epred_set_2EEMPTY\ ty_2Erealax_2Ereal))))\ (ap\ (ap\ (c_2Epred_set_2EINSERT \\
& ty_2Erealax_2Ereal)\ V1y)\ (c_2Epred_set_2EEMPTY\ ty_2Erealax_2Ereal)))) = \\
& (ap\ c_2Ereal_topology_2EDist\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal)\ V0x)\ V1y))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& ((ap\ c_2Ereal_topology_2Ehausdist\ (ap\ (ap\ (c_2Epair_2E_2C\ (\\
& \quad 2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))\ V0s)\ V1t))) = (\\
& \quad ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ ty_2Erealax_2Ereal)\ (ap\ (ap\ c_2Ebool_2E_2F_5C \\
& \quad \quad (ap\ c_2Ebool_2E_7E\ (ap\ (ap\ (c_2Emin_2E_3D\ (2^{ty_2Erealax_2Ereal})) \\
& (ap\ (ap\ (c_2Epred_set_2EUNION\ ty_2Erealax_2Ereal)\ (ap\ (c_2Epred_set_2EGSPEC \\
& \quad ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ (\lambda V2x \in ty_2Erealax_2Ereal. \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ 2)\ (ap\ c_2Ereal_topology_2Esetdist \\
& \quad \quad (ap\ (ap\ (c_2Epair_2E_2C\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal})) \\
& (ap\ (ap\ (c_2Epred_set_2EINSERT\ ty_2Erealax_2Ereal)\ V2x)\ (c_2Epred_set_2EEMPTY \\
& \quad ty_2Erealax_2Ereal))))\ V1t))))\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal \\
& \quad V2x)\ V0s))))\ (ap\ (c_2Epred_set_2EGSPEC\ ty_2Erealax_2Ereal \\
& \quad ty_2Erealax_2Ereal)\ (\lambda V3y \in ty_2Erealax_2Ereal.(ap\ (ap\ (c_2Epair_2E_2C \\
& \quad ty_2Erealax_2Ereal\ 2)\ (ap\ c_2Ereal_topology_2Esetdist\ (ap \\
& \quad \quad (ap\ (c_2Epair_2E_2C\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal})) \\
& (ap\ (ap\ (c_2Epred_set_2EINSERT\ ty_2Erealax_2Ereal)\ V3y)\ (c_2Epred_set_2EEMPTY \\
& \quad ty_2Erealax_2Ereal))))\ V0s))))\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal \\
& \quad V3y)\ V1t))))\ (c_2Epred_set_2EEMPTY\ ty_2Erealax_2Ereal)))) \\
& \quad (ap\ (c_2Ebool_2E_3F\ ty_2Erealax_2Ereal)\ (\lambda V4b \in ty_2Erealax_2Ereal. \\
& \quad (ap\ (c_2Ebool_2E_21\ ty_2Erealax_2Ereal)\ (\lambda V5d \in ty_2Erealax_2Ereal. \\
& \quad (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal \\
& \quad \quad V5d)\ (ap\ (ap\ (c_2Epred_set_2EUNION\ ty_2Erealax_2Ereal)\ (ap\ (\\
& \quad \quad \quad c_2Epred_set_2EGSPEC\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal) \\
& \quad \quad (\lambda V6x \in ty_2Erealax_2Ereal.(ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal \\
& \quad \quad \quad 2)\ (ap\ c_2Ereal_topology_2Esetdist\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad \quad \quad (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))\ (ap\ (ap\ (c_2Epred_set_2EINSERT \\
& \quad \quad \quad ty_2Erealax_2Ereal)\ V6x)\ (c_2Epred_set_2EEMPTY\ ty_2Erealax_2Ereal)))) \\
& \quad \quad \quad V1t))))\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ V6x)\ V0s)))) \\
& \quad \quad (ap\ (c_2Epred_set_2EGSPEC\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal) \\
& \quad \quad (\lambda V7y \in ty_2Erealax_2Ereal.(ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal \\
& \quad \quad \quad 2)\ (ap\ c_2Ereal_topology_2Esetdist\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad \quad \quad (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))\ (ap\ (ap\ (c_2Epred_set_2EINSERT \\
& \quad \quad \quad ty_2Erealax_2Ereal)\ V7y)\ (c_2Epred_set_2EEMPTY\ ty_2Erealax_2Ereal)))) \\
& \quad \quad \quad V0s))))\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ V7y)\ V1t)))) \\
& \quad \quad \quad (ap\ (ap\ c_2Ereal_2Ereal_lte\ V5d)\ V4b))))\ (ap\ c_2Ereal_2Esup \\
& (ap\ (ap\ (c_2Epred_set_2EUNION\ ty_2Erealax_2Ereal)\ (ap\ (c_2Epred_set_2EGSPEC \\
& \quad ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ (\lambda V8x \in ty_2Erealax_2Ereal. \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ 2)\ (ap\ c_2Ereal_topology_2Esetdist \\
& \quad \quad (ap\ (ap\ (c_2Epair_2E_2C\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal})) \\
& (ap\ (ap\ (c_2Epred_set_2EINSERT\ ty_2Erealax_2Ereal)\ V8x)\ (c_2Epred_set_2EEMPTY \\
& \quad ty_2Erealax_2Ereal))))\ V1t))))\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal \\
& \quad V8x)\ V0s))))\ (ap\ (c_2Epred_set_2EGSPEC\ ty_2Erealax_2Ereal \\
& \quad ty_2Erealax_2Ereal)\ (\lambda V9y \in ty_2Erealax_2Ereal.(ap\ (ap\ (c_2Epair_2E_2C \\
& \quad ty_2Erealax_2Ereal\ 2)\ (ap\ c_2Ereal_topology_2Esetdist\ (ap \\
& \quad \quad (ap\ (c_2Epair_2E_2C\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal})) \\
& (ap\ (ap\ (c_2Epred_set_2EINSERT\ ty_2Erealax_2Ereal)\ V9y)\ (c_2Epred_set_2EEMPTY \\
& \quad ty_2Erealax_2Ereal))))\ V0s))))\ (\&ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal \\
& \quad V9y)\ V1t))))\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)))) \\
& \quad \quad \quad (47)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (48)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (49)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (50)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (51)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (52)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (53)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \quad (54)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (57)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (\forall V3s \in \\
& 2. (((p V0p) \Leftrightarrow (p (ap (ap (ap (c_2Ebool_2ECOND 2) V1q) V2r) V3s))) \Leftrightarrow \\
& (((p V0p) \vee ((p V1q) \vee (\neg(p V3s)))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge \\
& (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V3s)))) \wedge (((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))) \wedge ((p V1q) \vee ((p V3s) \vee (\neg(p V0p))))))))))))) \\
& \tag{58}
\end{aligned}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{59}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{60}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{61}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{62}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{63}$$

Theorem 1

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap c_2Ereal_topology_2Ehausdist (ap (ap (c_2Epair_2E_2C (\\
& 2^{ty_2Erealax_2Ereal} (2^{ty_2Erealax_2Ereal})) (ap (ap (c_2Epred_set_2EINSERT \\
& ty_2Erealax_2Ereal) V0x) (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal))) \\
& (ap (ap (c_2Epred_set_2EINSERT ty_2Erealax_2Ereal) V1y) (c_2Epred_set_2EEMPTY \\
& ty_2Erealax_2Ereal)))))) = (ap c_2Ereal_topology_2EDist (ap (\\
& ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0x) \\
& V1y))))))
\end{aligned}$$