

thm_2Ereal__topology_2EHAUSDIST__SYM
(TMNjGweUoGsjEkDTvg-
pxDM9xodedU3MM64H)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 3 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \tag{4}$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}) \tag{5}$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{6}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{7}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (8)$$

Definition 4 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p (ap\ P\ x))$ then $(the\ (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}$

Definition 6 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ ty_2Ereal$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal)}) \quad (9)$$

Definition 7 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2$

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ (2^{A_27a}$

Definition 11 We define c_2Ereal_2Esup to be $\lambda V0P \in (2^{ty_2Erealax_2Ereal}).(ap\ (c_2Emin_2E_40\ ty_2Ereal$

Definition 12 We define $c_2Ebool_2E_EF$ to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 13 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_EF$

Definition 14 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 15 We define $c_2Ebool_2E_EIN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x))$

Definition 16 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E_EF)$.

Definition 17 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (10)$$

Definition 18 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (11)$$

Definition 19 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2Ereal_topology_2Esetdist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2Esetdist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod (2^{ty_2Erealax_2Ereal}) (2^{ty_2Erealax_2Ereal})}))) \quad (12)$$

Definition 20 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2E$

Definition 21 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(\lambda V$

Let $c_2Ereal_topology_2Ehausdist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2Ehausdist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod (2^{ty_2Erealax_2Ereal}) (2^{ty_2Erealax_2Ereal})}))) \quad (13)$$

Assume the following.

$$True \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1t \in (2^{A_27a}).((ap (ap (c_2Epred_set_2EUNION A_27a) V0s) V1t) = (ap (ap (c_2Epred_set_2EUNION A_27a) V1t) V0s)))) \quad (16)$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& ((ap\ c_2Ereal_topology_2Ehausdist\ (ap\ (ap\ (c_2Epair_2E_2C\ (\\
& \quad 2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))\ V0s)\ V1t))) = (\\
& ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ ty_2Erealax_2Ereal)\ (ap\ (ap\ c_2Ebool_2E_2F_5C \\
& \quad (ap\ c_2Ebool_2E_7E\ (ap\ (ap\ (c_2Emin_2E_3D\ (2^{ty_2Erealax_2Ereal})) \\
& (ap\ (ap\ (c_2Epred_set_2EUNION\ ty_2Erealax_2Ereal)\ (ap\ (c_2Epred_set_2EGSPEC \\
& \quad ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ (\lambda V2x \in ty_2Erealax_2Ereal. \\
& (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ 2)\ (ap\ c_2Ereal_topology_2Esetdist \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal})) \\
& (ap\ (ap\ (c_2Epred_set_2EINSERT\ ty_2Erealax_2Ereal)\ V2x)\ (c_2Epred_set_2EEMPTY \\
& \quad ty_2Erealax_2Ereal))))\ V1t))))\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal \\
& \quad V2x)\ V0s))))\ (ap\ (c_2Epred_set_2EGSPEC\ ty_2Erealax_2Ereal \\
& \quad ty_2Erealax_2Ereal)\ (\lambda V3y \in ty_2Erealax_2Ereal.(ap\ (ap\ (c_2Epair_2E_2C \\
& \quad ty_2Erealax_2Ereal\ 2)\ (ap\ c_2Ereal_topology_2Esetdist\ (ap \\
& \quad (ap\ (c_2Epair_2E_2C\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal})) \\
& (ap\ (ap\ (c_2Epred_set_2EINSERT\ ty_2Erealax_2Ereal)\ V3y)\ (c_2Epred_set_2EEMPTY \\
& \quad ty_2Erealax_2Ereal))))\ V0s))))\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal \\
& \quad V3y)\ V1t))))\ (c_2Epred_set_2EEMPTY\ ty_2Erealax_2Ereal)))) \\
& \quad (ap\ (c_2Ebool_2E_3F\ ty_2Erealax_2Ereal)\ (\lambda V4b \in ty_2Erealax_2Ereal. \\
& \quad (ap\ (c_2Ebool_2E_21\ ty_2Erealax_2Ereal)\ (\lambda V5d \in ty_2Erealax_2Ereal. \\
& \quad (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal \\
& \quad V5d)\ (ap\ (ap\ (c_2Epred_set_2EUNION\ ty_2Erealax_2Ereal)\ (ap\ (\\
& \quad c_2Epred_set_2EGSPEC\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal) \\
& \quad (\lambda V6x \in ty_2Erealax_2Ereal.(ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal \\
& \quad 2)\ (ap\ c_2Ereal_topology_2Esetdist\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))\ (ap\ (ap\ (c_2Epred_set_2EINSERT \\
& \quad ty_2Erealax_2Ereal)\ V6x)\ (c_2Epred_set_2EEMPTY\ ty_2Erealax_2Ereal)))) \\
& \quad V1t))))\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ V6x)\ V0s)))) \\
& \quad (ap\ (c_2Epred_set_2EGSPEC\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal) \\
& \quad (\lambda V7y \in ty_2Erealax_2Ereal.(ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal \\
& \quad 2)\ (ap\ c_2Ereal_topology_2Esetdist\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))\ (ap\ (ap\ (c_2Epred_set_2EINSERT \\
& \quad ty_2Erealax_2Ereal)\ V7y)\ (c_2Epred_set_2EEMPTY\ ty_2Erealax_2Ereal)))) \\
& \quad V0s))))\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ V7y)\ V1t))))\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ V7y)\ V1t))))\ (ap\ (ap\ (c_2Ereal_2Ereal_lte\ V5d)\ V4b))))\ (ap\ c_2Ereal_2Esup \\
& (ap\ (ap\ (c_2Epred_set_2EUNION\ ty_2Erealax_2Ereal)\ (ap\ (c_2Epred_set_2EGSPEC \\
& \quad ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ (\lambda V8x \in ty_2Erealax_2Ereal. \\
& (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ 2)\ (ap\ c_2Ereal_topology_2Esetdist \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal})) \\
& (ap\ (ap\ (c_2Epred_set_2EINSERT\ ty_2Erealax_2Ereal)\ V8x)\ (c_2Epred_set_2EEMPTY \\
& \quad ty_2Erealax_2Ereal))))\ V1t))))\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal \\
& \quad V8x)\ V0s))))\ (ap\ (c_2Epred_set_2EGSPEC\ ty_2Erealax_2Ereal \\
& \quad ty_2Erealax_2Ereal)\ (\lambda V9y \in ty_2Erealax_2Ereal.(ap\ (ap\ (c_2Epair_2E_2C \\
& \quad ty_2Erealax_2Ereal\ 2)\ (ap\ c_2Ereal_topology_2Esetdist\ (ap \\
& \quad (ap\ (c_2Epair_2E_2C\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal})) \\
& (ap\ (ap\ (c_2Epred_set_2EINSERT\ ty_2Erealax_2Ereal)\ V9y)\ (c_2Epred_set_2EEMPTY \\
& \quad ty_2Erealax_2Ereal))))\ V0s))))\ (\wp\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal \\
& \quad V9y)\ V1t))))\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)))) \\
& \quad (17)
\end{aligned}$$

Theorem 1

$$\begin{aligned} & (\forall V0s \in (2^{ty_2Erealax_2Ereal}). (\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\ & ((ap\ c_2Ereal_topology_2Ehausdist\ (ap\ (ap\ (c_2Epair_2E_2C\ (\\ & \quad 2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))\ V0s)\ V1t)) = (\\ & ap\ c_2Ereal_topology_2Ehausdist\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{ty_2Erealax_2Ereal}) \\ & \quad (2^{ty_2Erealax_2Ereal}))\ V1t)\ V0s)))))) \end{aligned}$$