

thm_2Ereal__topology_2EHOMEOMORPHIC__INJECTIVE__LINE
(TM-
NTvwakNH13RSUeyyL1bBwAeGx2hUdeszH)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_7E$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 4 We define $c_2Ebool_2E_5C$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 7 We define $c_2Ebool_2E_IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (2)$$

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2E_2F_5C$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (3)$$

Definition 10 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (4)$$

Let $c_2Ereal_topology_2Ehomeomorphism : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2Ehomeomorphism \in ((2^{(ty_2Epair_2Eprod\ (ty_2Erealax_2Ereal)^{ty_2Erealax_2Ereal})\ (ty_2Erealax_2Ereal)})^{(ty_2Erealax_2Ereal)}) \quad (5)$$

Definition 11 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p\ (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p\ x)$ of type $\iota \Rightarrow \iota$).

Definition 12 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ P))))$

Definition 13 We define $c_2Ereal_topology_2Ehomeomorphic$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).\lambda V1t \in (2^{ty_2Erealax_2Ereal})$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (6)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}\ ty_2Erealax_2Ereal)^{ty_2Erealax_2Ereal}) \quad (7)$$

Definition 14 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ (c_2Erealax_2Ereal_REP_CLASS\ a)))$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Erealax_2Ereal})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)} \quad (8)$$

Let $c_2Erealax_2Etrealeq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealeq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}\ ty_2Erealax_2Ereal)^{ty_2Erealax_2Ereal})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)} \quad (9)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}} \quad (10)$$

Definition 15 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 16 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$. Let $c_2Erealax_2Ereal_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}))^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)} \quad (11)$$

Definition 17 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$.

Definition 18 We define $c_2Ereal_topology_2Elinear$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})$.

Assume the following.

$$True \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (14)$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1t \in (2^{ty_2Erealax_2Ereal}).((p\ (ap\ (ap\ c_2Ereal_topology_2Ehomeomorphic\ V0s)\ V1t)) \Leftrightarrow (p\ (ap\ (ap\ c_2Ereal_topology_2Ehomeomorphic\ V1t)\ V0s)))))) \quad (15)$$

Assume the following.

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in (2^{ty_2Erealax_2Ereal}).(\forall V2t \in (2^{ty_2Erealax_2Ereal}).(((p\ (ap\ c_2Ereal_topology_2Elinear\ V0f)) \wedge (\forall V3x \in ty_2Erealax_2Ereal.(\forall V4y \in ty_2Erealax_2Ereal.(((ap\ V0f\ V3x) = (ap\ V0f\ V4y)) \Rightarrow (V3x = V4y)))))) \Rightarrow ((p\ (ap\ (ap\ c_2Ereal_topology_2Ehomeomorphic\ V1s)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ V0f)\ V1s))\ V2t)) \Leftrightarrow (p\ (ap\ (ap\ c_2Ereal_topology_2Ehomeomorphic\ V1s)\ V2t)))))) \quad (16)$$

Theorem 1

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in (2^{ty_2Erealax_2Ereal}).(\forall V2t \in (2^{ty_2Erealax_2Ereal}).(((p\ (ap\ c_2Ereal_topology_2Elinear\ V0f)) \wedge (\forall V3x \in ty_2Erealax_2Ereal.(\forall V4y \in ty_2Erealax_2Ereal.(((ap\ V0f\ V3x) = (ap\ V0f\ V4y)) \Rightarrow (V3x = V4y)))))) \Rightarrow ((p\ (ap\ (ap\ c_2Ereal_topology_2Ehomeomorphic\ V1s)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ V0f)\ V2t)) \Leftrightarrow (p\ (ap\ (ap\ c_2Ereal_topology_2Ehomeomorphic\ V1s)\ V2t)))))) \quad (16)$$