

thm_2Ereal_topology_2EHOMEOMORPHISM_FROM_COMPOS
(TMQmqKeSZtTnQNpT-
sqybH2cCySKrtnd5RMy)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V$

Definition 4 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 5 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a})$

Definition 6 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a}) A_27a$

Definition 7 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})$

Definition 8 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in (A_27b^{A_27c}).\lambda V1g$

Definition 9 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 10 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$
of type ι .

Definition 11 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap ($

Definition 12 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \quad (1)$$

Definition 13 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (2)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (3)$$

Definition 14 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ x\ y)$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (4)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (5)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (6)$$

Definition 15 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ ty_2Erealax_2Ereal)\ a)$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (7)$$

Definition 16 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.(c_2Erealax_2Etreall_lt\ T1\ T2)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (8)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (10)$$

Definition 17 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (11)$$

Definition 18 We define $c_Ereal_topology_Econtinuous_on$ to be $\lambda V0f \in (ty_Erealax_Ereal^{ty_Ereal})$

Let $c_Epred_set_EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epred_set_EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_Epair_Eprod\ A_27a\ 2)^{A_27b}}) \quad (12)$$

Definition 19 We define $c_Epred_set_EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 20 We define $c_Ereal_topology_EOpen$ to be $\lambda V0s \in (2^{ty_Erealax_Ereal}).(ap\ (c_Ebool_E2$

Let $ty_Etopology_Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_Etopology_Etopology\ A0) \quad (13)$$

Let $c_Etopology_Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_Etopology_Etopology\ A_27a \in ((ty_Etopology_Etopology\ A_27a)^{(2^{(2^{A_27a})})}) \quad (14)$$

Definition 21 We define $c_Ereal_topology_Eeuclidean$ to be $(ap\ (c_Etopology_Etopology\ ty_Erealax$

Let $c_Etopology_Eopen_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_Etopology_Eopen_in\ A_27a \in ((2^{(2^{A_27a})})^{(ty_Etopology_Etopology\ A_27a)}) \quad (15)$$

Definition 22 We define $c_Epred_set_EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c$

Definition 23 We define $c_Ereal_topology_Esubtopology$ to be $\lambda A_27a : \iota.\lambda V0top \in (ty_Etopology_Etopology$

Let $c_Ereal_topology_Ehomeomorphism : \iota$ be given. Assume the following.

$$c_Ereal_topology_Ehomeomorphism \in ((2^{(ty_Epair_Eprod\ (ty_Erealax_Ereal^{ty_Erealax_Ereal})\ (ty_Erealax_Ereal^{ty_Erealax_Ereal}))}) \quad (16)$$

Definition 24 We define c_Ebool_E2E to be $(ap\ (c_Ebool_E2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 25 We define $c_Ebool_E2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_Ebool_E2E_21\ 2)\ (\lambda V2t \in$

Definition 26 We define $c_Ebool_E2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_Emin_E3D_3D_3E\ V0t)\ c_Ebool_E2E$

Assume the following.

$$True \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (22) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (23) \end{aligned}$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \quad (24) \end{aligned}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p\ V0t)))))) \quad (28) \end{aligned}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((p V0P) \wedge (\forall V2x \in A.27a. (p (ap V1Q V2x)))))) \Leftrightarrow (\forall V3x \in A.27a. ((p V0P) \wedge (p (ap V1Q V3x)))))) \quad (29)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p (ap V1P V2x)) \vee (p V0Q)))))) \Leftrightarrow ((\forall V3x \in A.27a. (p (ap V1P V3x))) \vee (p V0Q)))) \quad (30)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p V0P) \vee (p (ap V1Q V2x)))))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a. (p (ap V1Q V3x)))))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B)))))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (35)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (36)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x.27 \in 2. (\forall V2y \in 2. (\forall V3y.27 \in 2. (((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \quad (37)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27a^{A_27c}). \\
& (\forall V2x \in A_27c. ((ap\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27c\ A_27b\ A_27a) \\
& V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Ecombin_2EI \\
A_27a)\ V0x) = V0x)) \tag{39}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\
& (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& A_27a)\ V2x)\ V0s) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b^{A_27a}). \\
& ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\
& A_27a\ A_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\
& (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27c^{A_27b}). (\forall V1g \in (A_27b^{A_27a}). \\
& (\forall V2s \in (2^{A_27a}). ((ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_27a \\
& A_27c)\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27a\ A_27c\ A_27b)\ V0f)\ V1g))\ V2s) = \\
& (ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_27b\ A_27c)\ V0f)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\
& A_27a\ A_27b)\ V1g)\ V2s))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V1s \in \\
& (2^{ty_2Erealax_2Ereal}). (\forall V2t \in (2^{ty_2Erealax_2Ereal}). \\
& (((p\ (ap\ (ap\ c_2Ereal_topology_2Econtinuous_on\ V0f)\ V1s)) \wedge \\
& (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ ty_2Erealax_2Ereal)\ V2t)\ V1s))) \Rightarrow \\
& (p\ (ap\ (ap\ c_2Ereal_topology_2Econtinuous_on\ V0f)\ V2t))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1g \in \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V2s \in (2^{ty_2Erealax_2Ereal}). \\
& ((p (ap (ap (ap c_2Ereal_topology_2Econtinuous_on V0f) V2s)) \wedge \\
& (p (ap (ap (ap c_2Ereal_topology_2Econtinuous_on V1g) (ap (ap (c_2Epred_set_2EIMAGE \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0f) V2s)))) \Rightarrow (p (ap (\\
& ap c_2Ereal_topology_2Econtinuous_on (ap (ap (c_2Ecombin_2Eo \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V1g) V0f)) V2s))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1g \in \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V2s \in (2^{ty_2Erealax_2Ereal}). \\
& (\forall V3t \in (2^{ty_2Erealax_2Ereal}).(\forall V4u \in (2^{ty_2Erealax_2Ereal}). \\
& (((p (ap (ap c_2Ereal_topology_2Econtinuous_on V0f) V2s)) \wedge \\
& (((ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V0f) V2s) = V3t) \wedge ((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) \\
& (ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V1g) V3t)) V4u)) \wedge (\forall V5k \in (2^{ty_2Erealax_2Ereal}).((p (ap \\
& (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& ty_2Erealax_2Ereal) c_2Ereal_topology_2Eeuclidean) V2s)) \\
& V5k)) \Rightarrow (p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) \\
& (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& c_2Ereal_topology_2Eeuclidean) V4u)) (ap (ap (c_2Epred_set_2EIMAGE \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) (ap (ap (c_2Ecombin_2Eo \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V1g) V0f)) V5k)))))) \Rightarrow (\forall V6k \in (2^{ty_2Erealax_2Ereal}). \\
& ((p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) (ap \\
& (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& c_2Ereal_topology_2Eeuclidean) V3t)) V6k)) \Rightarrow (p (ap (ap (c_2Etopology_2Eopen_in \\
& ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& ty_2Erealax_2Ereal) c_2Ereal_topology_2Eeuclidean) V4u)) \\
& (ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V1g) V6k)))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& (\forall V2f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V3g \in \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).((p (ap (ap c_2Ereal_topology_2Ehomeomorphism \\
& (ap (ap (c_2Epair_2E_2C (2^{ty_2Erealax_2Ereal}) (2^{ty_2Erealax_2Ereal})) \\
& V0s) V1t)) (ap (ap (c_2Epair_2E_2C (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal} \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})) V2f) V3g))) \Leftrightarrow ((\forall V4x \in \\
& ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V4x) V0s)) \Rightarrow ((ap V3g (ap V2f V4x)) = V4x))) \wedge (((ap (ap (c_2Epred_set_2EIMAGE \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V2f) V0s) = V1t) \wedge ((p (\\
& ap (ap c_2Ereal_topology_2Econtinuous_on V2f) V0s)) \wedge ((\forall V5y \in \\
& ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V5y) V1t)) \Rightarrow ((ap V2f (ap V3g V5y)) = V5y))) \wedge (((ap (ap (c_2Epred_set_2EIMAGE \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V3g) V1t) = V0s) \wedge (p (ap \\
& (ap c_2Ereal_topology_2Econtinuous_on V3g) V1t))))))))))))) \\
& \tag{46}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in \\
& (2^{ty_2Erealax_2Ereal}).(\forall V2t \in (2^{ty_2Erealax_2Ereal}). \\
& (((p (ap (ap c_2Ereal_topology_2Econtinuous_on V0f) V1s)) \wedge \\
& (((ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V0f) V1s) = V2t) \wedge ((\forall V3x \in ty_2Erealax_2Ereal.(\forall V4y \in \\
& ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V3x) V1s)) \wedge ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V4y) \\
& V1s)) \wedge ((ap V0f V3x) = (ap V0f V4y)))) \Rightarrow (V3x = V4y)))) \wedge ((\forall V5u \in \\
& (2^{ty_2Erealax_2Ereal}).((p (ap (ap (c_2Etopology_2Eopen_in \\
& ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& ty_2Erealax_2Ereal) c_2Ereal_topology_2Eeuclidean) V1s)) \\
& V5u)) \Rightarrow (p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) \\
& (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& c_2Ereal_topology_2Eeuclidean) V2t)) (ap (ap (c_2Epred_set_2EIMAGE \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0f) V5u))))))))) \Rightarrow (\exists V6g \in \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(p (ap (ap c_2Ereal_topology_2Ehomeomorphism \\
& (ap (ap (c_2Epair_2E_2C (2^{ty_2Erealax_2Ereal}) (2^{ty_2Erealax_2Ereal})) \\
& V1s) V2t)) (ap (ap (c_2Epair_2E_2C (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal} \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})) V0f) V6g))))))))) \\
& \tag{47}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1g \in \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V2s \in (2^{ty_2Erealax_2Ereal}). \\
& (\forall V3t \in (2^{ty_2Erealax_2Ereal}).(\forall V4u \in (2^{ty_2Erealax_2Ereal}). \\
& (((p (ap (ap (ap c_2Ereal_topology_2Ehomeomorphism (ap (ap (c_2Epair_2E_2C \\
& (2^{ty_2Erealax_2Ereal}) (2^{ty_2Erealax_2Ereal})) V2s) V3t)) (\\
& ap (ap (c_2Epair_2E_2C (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal} \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})) V0f) V1g))) \wedge (p (ap \\
& (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& ty_2Erealax_2Ereal) c_2Ereal_topology_2Eeuclidean) V2s)) \\
& V4u))) \Rightarrow (p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) \\
& (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& c_2Ereal_topology_2Eeuclidean) V3t)) (ap (ap (c_2Epred_set_2EIMAGE \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0f) V4u))))))))) \\
& \tag{48}
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{49}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \\
& \tag{51}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \\
& \tag{52}
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \\
& \tag{54}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \\
& \tag{55}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow (\\
& (p \vee 1q) \vee (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee \neg(p \vee 1q)) \wedge ((p \vee 0p) \vee \neg(p \vee 2r))) \wedge \\
& ((p \vee 1q) \vee ((p \vee 2r) \vee \neg(p \vee 0p))))))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow (\\
& (p \vee 1q) \Rightarrow (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee (p \vee 1q)) \wedge ((p \vee 0p) \vee \neg(p \vee 2r))) \wedge ((\\
& \neg(p \vee 1q) \vee ((p \vee 2r) \vee \neg(p \vee 0p))))))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \vee 0p) \Leftrightarrow \neg(p \vee 1q))) \Leftrightarrow (((p \vee 0p) \vee \\
& (p \vee 1q)) \wedge (\neg(p \vee 1q) \vee \neg(p \vee 0p))))))
\end{aligned} \tag{58}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \vee 0p) \Rightarrow (p \vee 1q))) \Rightarrow (p \vee 0p))) \tag{59}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \vee 0p) \Rightarrow (p \vee 1q))) \Rightarrow \neg(p \vee 1q))) \tag{60}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \vee 0p) \vee (p \vee 1q))) \Rightarrow \neg(p \vee 0p))) \tag{61}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \vee 0p) \vee (p \vee 1q))) \Rightarrow \neg(p \vee 1q))) \tag{62}$$

Assume the following.

$$(\forall V0p \in 2. (\neg(\neg(p \vee 0p))) \Rightarrow (p \vee 0p)) \tag{63}$$

Theorem 1

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1g \in \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V2s \in (2^{ty_2Erealax_2Ereal}). \\
& (\forall V3t \in (2^{ty_2Erealax_2Ereal}).(\forall V4u \in (2^{ty_2Erealax_2Ereal}). \\
& ((p (ap (ap c_2Ereal_topology_2Econtinuous_on V0f) V2s)) \wedge \\
& (((ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V0f) V2s) = V3t) \wedge ((p (ap (ap c_2Ereal_topology_2Econtinuous_on \\
& V1g) V3t)) \wedge ((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) \\
& (ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V1g) V3t)) V4u)) \wedge (\exists V5h \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). \\
& (p (ap (ap c_2Ereal_topology_2Ehomeomorphism (ap (ap (c_2Epair_2E_2C \\
& (2^{ty_2Erealax_2Ereal} (2^{ty_2Erealax_2Ereal})) V2s) V4u)) (\\
& ap (ap (c_2Epair_2E_2C (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal} \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})) (ap (ap (c_2Ecombin_2Eo \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V1g) V0f)) V5h)))))) \Rightarrow ((\exists V6f_27 \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). \\
& (p (ap (ap c_2Ereal_topology_2Ehomeomorphism (ap (ap (c_2Epair_2E_2C \\
& (2^{ty_2Erealax_2Ereal} (2^{ty_2Erealax_2Ereal})) V2s) V3t)) (\\
& ap (ap (c_2Epair_2E_2C (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal} \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})) V0f) V6f_27)))) \wedge (\\
& \exists V7g_27 \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(p \\
& (ap (ap c_2Ereal_topology_2Ehomeomorphism (ap (ap (c_2Epair_2E_2C \\
& (2^{ty_2Erealax_2Ereal} (2^{ty_2Erealax_2Ereal})) V3t) V4u)) (\\
& ap (ap (c_2Epair_2E_2C (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal} \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})) V1g) V7g_27)))))))))
\end{aligned}$$