

thm\_2Ereal\_\_topology\_2EHOMEOMORPHISM\_\_IMP\_\_CLOSED\_\_M  
(TMd9aN5VngSwEFFmKyuzxXbsMusy2FWH55p)

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**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A$ . **if**  $(\exists x \in A.p (ap P x))$  **then**  $(the (\lambda x.x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (1)$$

**Definition 4** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a P))))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \quad (2)$$

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty ty\_2Erealx\_2Ereal \quad (3)$$

**Definition 5** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 6** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 7** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a} P))))$

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (lambda V2t \in 2.V2t))))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}})$$
 (4)

**Definition 9** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Let  $c\_2Ereal\_topology\_2EDist : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)})$$
 (5)

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal$$
 (6)

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal})$$
 (7)

**Definition 10** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E\_40\ (t$

Let  $c\_2Erealax\_2Etreallt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreallt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)})$$
 (8)

**Definition 11** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega$$
 (9)

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum$$
 (10)

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega})$$
 (11)

**Definition 12** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})$$
 (12)

**Definition 13** We define  $c\_2Ereal\_topology\_2EOpen$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap\ (c\_2Ebool\_2E\_2$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (13)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c\_2Etopology\_2Etopology\ A.27a \in ((ty\_2Etopology\_2Etopology\ A.27a)^{(2^{(2^A-27a)})}) \quad (14)$$

**Definition 14** We define  $c\_2Ereal\_topology\_2Eeuclidean$  to be  $(ap\ (c\_2Etopology\_2Etopology\ ty\_2Erealax\_2Ereal))$ .

Let  $c\_2Etopology\_2Eopen\_in : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c\_2Etopology\_2Eopen\_in\ A.27a \in ((2^{(2^A-27a)})^{(ty\_2Etopology\_2Etopology\ A.27a)}) \quad (15)$$

**Definition 15** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A.27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ (c\_2Etopology\_2Etopology\ ty\_2Erealax\_2Ereal))$ .

**Definition 16** We define  $c\_2Ereal\_topology\_2Esubtopology$  to be  $\lambda A.27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology\ ty\_2Erealax\_2Ereal)$ .

**Definition 17** We define  $c\_2Ereal\_topology\_2Econtinuous\_on$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})$ .

**Definition 18** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0f \in (A.27b^{A-27a}).\lambda V1s \in (2^{A-27a})$ .

Let  $c\_2Ereal\_topology\_2Ehomeomorphism : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2Ehomeomorphism \in ((2^{(ty\_2Epair\_2Eprod\ (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}\ ty\_2Erealax\_2Ereal))\ (ty\_2Erealax\_2Ereal))}) \quad (16)$$

**Definition 19** We define  $c\_2Ebool\_2E2F$  to be  $(ap\ (c\_2Ebool\_2E21\ 2))\ (\lambda V0t \in 2.V0t)$ .

**Definition 20** We define  $c\_2Ebool\_2E5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E21\ 2))\ (\lambda V2t \in 2.V2t)))$ .

**Definition 21** We define  $c\_2Ebool\_2E7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E21\ 2))$ .

**Definition 22** We define  $c\_2Epred\_set\_2E2DIFF$  to be  $\lambda A.27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ (c\_2Etopology\_2Etopology\ ty\_2Erealax\_2Ereal))$ .

**Definition 23** We define  $c\_2Epred\_set\_2E2BIGUNION$  to be  $\lambda A.27a : \iota.\lambda V0P \in (2^{(2^A-27a)}).(ap\ (c\_2Epred\_set\_2E2DIFF\ ty\_2Erealax\_2Ereal))$ .

**Definition 24** We define  $c\_2Etopology\_2E2topspace$  to be  $\lambda A.27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology\ ty\_2Erealax\_2Ereal)$ .

**Definition 25** We define  $c\_2Epred\_set\_2E2SUBSET$  to be  $\lambda A.27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ (c\_2Epred\_set\_2E2DIFF\ ty\_2Erealax\_2Ereal))$ .

**Definition 26** We define  $c\_2Etopology\_2E2closed\_in$  to be  $\lambda A.27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology\ ty\_2Erealax\_2Ereal)$ .

Assume the following.

$$True \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (21)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \quad (22)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (24)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p \ V0P) \wedge (\forall V2x \in A\_27a.(p \ (ap \ V1Q \ V2x)))) \Leftrightarrow (\forall V3x \in A\_27a.((p \ V0P) \wedge (p \ (ap \ V1Q \ V3x)))))) \quad (25)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p \ (ap \ V1P \ V2x)) \vee (p \ V0Q))) \Leftrightarrow ((\forall V3x \in A\_27a.(p \ (ap \ V1P \ V3x)) \vee (p \ V0Q)))))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V1B) \wedge (p \ V2C)) \vee (p \ V0A)) \Leftrightarrow (((p \ V1B) \vee (p \ V0A)) \wedge ((p \ V2C) \vee (p \ V0A)))))) \quad (27)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (28)$$

Assume the following.

$$2.((\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p \ V0x) \Leftrightarrow (p \ V1x_{.27})) \wedge ((p \ V1x_{.27}) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_{.27})))))) \Rightarrow ((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_{.27}) \Rightarrow (p \ V3y_{.27})))))) \Rightarrow \quad (29)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1a \in A_{.27a}.((\exists V2x \in A_{.27a}.((V2x = V1a) \wedge (p \ (ap \ V0P \ V2x)))) \Leftrightarrow (p \ (ap \ V0P \ V1a)))))) \quad (30)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty \ A_{.27b} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.(\forall V2a \in A_{.27a}.(\forall V3b \in A_{.27b}.(((ap \ (ap \ (c_{.2Epair_{.2E_{.2C}} \ A_{.27a} \ A_{.27b}) \ V0x) \ V1y) = (ap \ (ap \ (c_{.2Epair_{.2E_{.2C}} \ A_{.27a} \ A_{.27b}) \ V2a) \ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \quad (31)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in (2^{A_{.27a}}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_{.27a}.((p \ (ap \ (ap \ (c_{.2Ebool_{.2EIN}} \ A_{.27a}) \ V2x) \ V0s)) \Leftrightarrow (p \ (ap \ (ap \ (c_{.2Ebool_{.2EIN}} \ A_{.27a}) \ V2x) \ V1t))))))) \quad (32)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty \ A_{.27b} \Rightarrow (\forall V0f \in ((ty_{.2Epair_{.2Eprod}} \ A_{.27a} \ 2)^{A_{.27b}}).(\forall V1v \in A_{.27a}.((p \ (ap \ (ap \ (c_{.2Ebool_{.2EIN}} \ A_{.27a}) \ V1v) \ (ap \ (c_{.2Epred_{.set_{.2EGSPEC}} \ A_{.27a} \ A_{.27b}) \ V0f)))) \Leftrightarrow (\exists V2x \in A_{.27b}.((ap \ (ap \ (c_{.2Epair_{.2E_{.2C}} \ A_{.27a} \ 2) \ V1v) \ c_{.2Ebool_{.2ET}} = (ap \ V0f \ V2x))))))) \quad (33)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty \ A_{.27b} \Rightarrow (\forall V0y \in A_{.27b}.(\forall V1s \in (2^{A_{.27a}}).(\forall V2f \in (A_{.27b}^{A_{.27a}}).((p \ (ap \ (ap \ (c_{.2Ebool_{.2EIN}} \ A_{.27b}) \ V0y) \ (ap \ (ap \ (c_{.2Epred_{.set_{.2EIMAGE}} \ A_{.27a} \ A_{.27b}) \ V2f) \ V1s)))) \Leftrightarrow (\exists V3x \in A_{.27a}.((V0y = (ap \ V2f \ V3x)) \wedge (p \ (ap \ (ap \ (c_{.2Ebool_{.2EIN}} \ A_{.27a}) \ V3x) \ V1s))))))) \quad (34)$$

Assume the following.

$$(\forall V0s \in (2^{ty_{.2Erealax_{.2Ereal}}}).((ap \ (c_{.2Etopology_{.2Etopspace}} \ ty_{.2Erealax_{.2Ereal}}) \ (ap \ (ap \ (c_{.2Ereal_{.topology_{.2Esubtopology}} \ ty_{.2Erealax_{.2Ereal}}) \ c_{.2Ereal_{.topology_{.2Euclidean}}}) \ V0s)) = V0s)) \quad (35)$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1s \in \\
& \quad (2^{ty\_2Erealax\_2Ereal}).(\forall V2t \in (2^{ty\_2Erealax\_2Ereal}). \\
& \quad ((p (ap (ap (c\_2Ereal\_topology\_2Econtinuous\_on V0f) V1s)) \wedge \\
& \quad (p (ap (ap (c\_2Etopology\_2Eclosed\_in ty\_2Erealax\_2Ereal) (ap \\
& \quad (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\
& \quad c\_2Ereal\_topology\_2Eeuclidean) (ap (ap (c\_2Epred\_set\_2EIMAGE \\
& \quad ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) V0f) V1s))) V2t))) \Rightarrow ( \\
& \quad p (ap (ap (c\_2Etopology\_2Eclosed\_in ty\_2Erealax\_2Ereal) (ap \\
& \quad (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\
& \quad c\_2Ereal\_topology\_2Eeuclidean) V1s)) (ap (c\_2Epred\_set\_2EGSPEC \\
& \quad ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) (\lambda V3x \in ty\_2Erealax\_2Ereal. \\
& \quad (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal 2) V3x) (ap (ap c\_2Ebool\_2E\_2F\_5C \\
& \quad (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V3x) V1s)) (ap (ap ( \\
& \quad c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) (ap V0f V3x)) V2t)))))))))) \\
& \hspace{15em} (36)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1t \in (2^{ty\_2Erealax\_2Ereal}). \\
& \quad (\forall V2f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V3g \in \\
& \quad (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).((p (ap (ap c\_2Ereal\_topology\_2Ehomeomorphism \\
& \quad (ap (ap (c\_2Epair\_2E\_2C (2^{ty\_2Erealax\_2Ereal}) (2^{ty\_2Erealax\_2Ereal})) \\
& \quad V0s) V1t)) (ap (ap (c\_2Epair\_2E\_2C (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}) \\
& \quad (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})) V2f) V3g))) \Leftrightarrow ((\forall V4x \in \\
& \quad ty\_2Erealax\_2Ereal.((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\
& \quad V4x) V0s)) \Rightarrow ((ap V3g (ap V2f V4x)) = V4x))) \wedge (((ap (ap (c\_2Epred\_set\_2EIMAGE \\
& \quad ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) V2f) V0s) = V1t) \wedge ((p ( \\
& \quad ap (ap c\_2Ereal\_topology\_2Econtinuous\_on V2f) V0s)) \wedge ((\forall V5y \in \\
& \quad ty\_2Erealax\_2Ereal.((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\
& \quad V5y) V1t)) \Rightarrow ((ap V2f (ap V3g V5y)) = V5y))) \wedge (((ap (ap (c\_2Epred\_set\_2EIMAGE \\
& \quad ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) V3g) V1t) = V0s) \wedge (p (ap \\
& \quad (ap c\_2Ereal\_topology\_2Econtinuous\_on V3g) V1t)))))))))) \\
& \hspace{15em} (37)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (39)$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \\
& \hspace{15em} (40)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \\
& \hspace{15em} (41)
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (42)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg( \\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (( \\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (47)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (51)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (52)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in \\ & (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2s \in (2^{ty\_2Erealax\_2Ereal}). \\ & (\forall V3t \in (2^{ty\_2Erealax\_2Ereal}).(\forall V4u \in (2^{ty\_2Erealax\_2Ereal}). \\ & (((p (ap (ap c\_2Ereal\_topology\_2Ehomeomorphism (ap (ap (c\_2Epair\_2E\_2C \\ & (2^{ty\_2Erealax\_2Ereal}) (2^{ty\_2Erealax\_2Ereal})) V2s) V3t)) ( \\ & ap (ap (c\_2Epair\_2E\_2C (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}) \\ & (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})) V0f) V1g))) \wedge (p (ap \\ & (ap (c\_2Etopology\_2Eclosed\_in ty\_2Erealax\_2Ereal) (ap (ap ( \\ & c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Eeuclidean) \\ & V2s)) V4u))) \Rightarrow (p (ap (ap (c\_2Etopology\_2Eclosed\_in ty\_2Erealax\_2Ereal) \\ & (ap (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\ & c\_2Ereal\_topology\_2Eeuclidean) V3t)) (ap (ap (c\_2Epred\_set\_2EIMAGE \\ & ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) V0f) V4u))))))))) \end{aligned}$$