

thm\_2Ereal\_\_topology\_2EHOMEOMORPHISM\_\_INJECTIVE\_\_CLO  
(TMJd28mS1vo7i6TM83Xskk2SeuHhooKGMmb)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A$ . **if**  $(\exists x \in A.p (ap P x))$  **then** (the  $(\lambda x.x \in A \wedge p x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ecombin\_2E\_2K$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 5** We define  $c\_2Ecombin\_2E\_2S$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 6** We define  $c\_2Ecombin\_2E\_2I$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2E\_2S A\_27a (A\_27a^{A\_27a})) A\_27a))$

**Definition 7** We define  $c\_2Ebool\_2E\_2F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a P))))$

**Definition 8** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 9** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) P) P))$

**Definition 10** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{2}$$

**Definition 11** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Ereal\_2Ereal\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (3)$$

Let  $c\_2Ereal\_2Ereal\_2Ehomeomorphism : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_2Ehomeomorphism \in ((2^{(ty\_2Epair\_2Eprod (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}) (ty\_2Erealax\_2Ereal))}) \quad (4)$$

**Definition 12** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x))$

Let  $c\_2Ereal\_2Ereal\_2EDist : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal)}) \quad (5)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (6)$$

Let  $c\_2Erealax\_2Ereal\_2E\_2REP\_2E\_2CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_2E\_2REP\_2E\_2CLASS \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (7)$$

**Definition 13** We define  $c\_2Erealax\_2Ereal\_2E\_2REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E\_240 (t$

Let  $c\_2Erealax\_2Ereal\_2E\_2lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_2E\_2lt \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal)}) \quad (8)$$

**Definition 14** We define  $c\_2Erealax\_2Ereal\_2E\_2lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Let  $c\_2Enum\_2EZERO\_2E\_2REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_2E\_2REP \in \omega \quad (9)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (10)$$

Let  $c\_2Enum\_2EABS\_2E\_2num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_2E\_2num \in (ty\_2Enum\_2Enum^{\omega}) \quad (11)$$

**Definition 15** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_2E\_2num c\_2Enum\_2EZERO\_2E\_2REP)$ .

Let  $c\_2Ereal\_2Ereal\_2E\_2of\_2E\_2num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_2E\_2of\_2E\_2num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (12)$$

**Definition 16** We define  $c\_2Ereal\_topology\_2EOpen$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap (c\_2Ebool\_2E2$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Etopology\_2Etopology A0) \quad (13)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Etopology\_2Etopology A\_27a \in ((ty\_2Etopology\_2Etopology A\_27a)^{(2^{(2^A-27a)})}) \quad (14)$$

**Definition 17** We define  $c\_2Ereal\_topology\_2EEuclidean$  to be  $(ap (c\_2Etopology\_2Etopology ty\_2Erealax$

Let  $c\_2Etopology\_2Eopen\_in : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Etopology\_2Eopen\_in A\_27a \in ((2^{(2^A-27a)})^{(ty\_2Etopology\_2Etopology A\_27a)}) \quad (15)$$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A-27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A-27b}}) \quad (16)$$

**Definition 18** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap (c$

**Definition 19** We define  $c\_2Ereal\_topology\_2Esubtopology$  to be  $\lambda A\_27a : \iota. \lambda V0top \in (ty\_2Etopology\_2Etopology$

**Definition 20** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota. \lambda V0P \in (2^{(2^A-27a)}). (ap (c\_2Epred\_s$

**Definition 21** We define  $c\_2Etopology\_2Etopspace$  to be  $\lambda A\_27a : \iota. \lambda V0top \in (ty\_2Etopology\_2Etopology$

**Definition 22** We define  $c\_2Ebool\_2E21$  to be  $(ap (c\_2Ebool\_2E21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 23** We define  $c\_2Ebool\_2E7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E3D\_3D\_3E V0t) c\_2Ebool\_2E7E$

**Definition 24** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap (c\_2E$

**Definition 25** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap ($

**Definition 26** We define  $c\_2Etopology\_2Eclosed\_in$  to be  $\lambda A\_27a : \iota. \lambda V0top \in (ty\_2Etopology\_2Etopology$

**Definition 27** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A-27a}). \lambda V1s \in$

**Definition 28** We define  $c\_2Ereal\_topology\_2Econtinuous\_on$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Ereal$

**Definition 29** We define  $c\_2Ebool\_2E5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E21 2) (\lambda V2t \in$

Assume the following.

$$True \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (18)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (19)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (24)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (25)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \quad (26)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (27)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in A.27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t))))) \quad (29)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\neg(\forall V1x \in A.27a. (p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a. (\neg(p (ap V0P V2x))))) \quad (30)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\neg(\exists V1x \in A.27a. (p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a. (\neg(p (ap V0P V2x))))) \quad (31)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). (((p V0P) \wedge (\forall V2x \in A.27a. (p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A.27a. ((p V0P) \wedge (p (ap V1Q V3x))))) \quad (32)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in 2. (((\exists V2x \in A.27a. (p (ap V0P V2x))) \vee (p V1Q)) \Leftrightarrow (\exists V3x \in A.27a. ((p (ap V0P V3x)) \vee (p V1Q))))) \quad (33)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). (((p V0P) \vee (\exists V2x \in A.27a. (p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in A.27a. ((p V0P) \vee (p (ap V1Q V3x))))) \quad (34)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A.27a. (p (ap V1P V3x))) \vee (p V0Q)))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (39)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (40)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \quad (41)$$

Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow \forall A_{27b}.nonempty\ A_{27b} \Rightarrow (\forall V0P \in (2^{A_{27b}^{A_{27a}}}).((\forall V1x \in A_{27a}.(\exists V2y \in A_{27b}.(p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A_{27b}^{A_{27a}}).(\forall V4x \in A_{27a}.(p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x))))))) \quad (42)$$

Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow \forall A_{27b}.nonempty\ A_{27b} \Rightarrow (\forall V0f \in (A_{27b}^{A_{27a}}).(\forall V1s \in (2^{A_{27a}}).((\forall V2x \in A_{27a}.(\forall V3y \in A_{27a}.(((p\ (ap\ (ap\ (c_{2Ebool\_2EIN}\ A_{27a})\ V2x)\ V1s)) \wedge ((p\ (ap\ (ap\ (c_{2Ebool\_2EIN}\ A_{27a})\ V3y)\ V1s)) \wedge ((ap\ V0f\ V2x) = (ap\ V0f\ V3y)))) \Rightarrow (V2x = V3y)))) \Leftrightarrow (\exists V4g \in (A_{27a}^{A_{27b}}).(\forall V5x \in A_{27a}.((p\ (ap\ (ap\ (c_{2Ebool\_2EIN}\ A_{27a})\ V5x)\ V1s)) \Rightarrow ((ap\ V4g\ (ap\ V0f\ V5x)) = V5x))))))) \quad (43)$$

Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow (\forall V0x \in A_{27a}.((ap\ (c_{2Ecombin\_2EI}\ A_{27a})\ V0x) = V0x)) \quad (44)$$

Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow (\forall V0s \in (2^{A_{27a}}).(\forall V1t \in (2^{A_{27a}}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_{27a}.((p\ (ap\ (ap\ (c_{2Ebool\_2EIN}\ A_{27a})\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_{2Ebool\_2EIN}\ A_{27a})\ V2x)\ V1t)))))) \quad (45)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0y \in A\_27b. (\forall V1s \in (2^{A\_27a}). (\forall V2f \in (A\_27b^{A\_27a}). \\
& \quad ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27b)\ V0y)\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE \\
& \quad A\_27a\ A\_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A\_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\
& \quad (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V3x)\ V1s))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}). (\forall V1t \in (2^{ty\_2Erealax\_2Ereal}). \\
& \quad (\forall V2f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). (\forall V3g \in \\
& \quad (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). ((p\ (ap\ (ap\ c\_2Ereal\_topology\_2Ehomeomorphism \\
& \quad (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal})) \\
& \quad V0s)\ V1t))\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}) \\
& \quad (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}))\ V2f)\ V3g))) \Leftrightarrow ((\forall V4x \in \\
& \quad ty\_2Erealax\_2Ereal. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ ty\_2Erealax\_2Ereal) \\
& \quad V4x)\ V0s)) \Rightarrow ((ap\ V3g\ (ap\ V2f\ V4x)) = V4x)) \wedge (((ap\ (ap\ (c\_2Epred\_set\_2EIMAGE \\
& \quad ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V2f)\ V0s) = V1t) \wedge ((p\ ( \\
& \quad ap\ (ap\ c\_2Ereal\_topology\_2Econtinuous\_on\ V2f)\ V0s)) \wedge ((\forall V5y \in \\
& \quad ty\_2Erealax\_2Ereal. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ ty\_2Erealax\_2Ereal) \\
& \quad V5y)\ V1t)) \Rightarrow ((ap\ V2f\ (ap\ V3g\ V5y)) = V5y)) \wedge (((ap\ (ap\ (c\_2Epred\_set\_2EIMAGE \\
& \quad ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V3g)\ V1t) = V0s) \wedge (p\ (ap\ (ap\ c\_2Ereal\_topology\_2Econtinuous\_on\ V3g)\ V1t))))))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). (\forall V1g \in \\
& \quad (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). (\forall V2s \in (2^{ty\_2Erealax\_2Ereal}). \\
& \quad (\forall V3t \in (2^{ty\_2Erealax\_2Ereal}). (((p\ (ap\ (ap\ c\_2Ereal\_topology\_2Econtinuous\_on \\
& \quad V0f)\ V2s)) \wedge (((ap\ (ap\ (c\_2Epred\_set\_2EIMAGE\ ty\_2Erealax\_2Ereal \\
& \quad ty\_2Erealax\_2Ereal)\ V0f)\ V2s) = V3t) \wedge ((\forall V4x \in ty\_2Erealax\_2Ereal. \\
& \quad ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ ty\_2Erealax\_2Ereal)\ V4x)\ V2s)) \Rightarrow ((ap \\
& \quad V1g\ (ap\ V0f\ V4x)) = V4x)) \wedge (\forall V5u \in (2^{ty\_2Erealax\_2Ereal}). \\
& \quad ((p\ (ap\ (ap\ (c\_2Etopology\_2Eclosed\_in\ ty\_2Erealax\_2Ereal)\ ( \\
& \quad ap\ (ap\ (c\_2Ereal\_topology\_2Esubtopology\ ty\_2Erealax\_2Ereal) \\
& \quad c\_2Ereal\_topology\_2Eeuclidean)\ V2s))\ V5u)) \Rightarrow (p\ (ap\ (ap\ (c\_2Etopology\_2Eclosed\_in \\
& \quad ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (c\_2Ereal\_topology\_2Esubtopology \\
& \quad ty\_2Erealax\_2Ereal)\ c\_2Ereal\_topology\_2Eeuclidean)\ V3t)) \\
& \quad (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal) \\
& \quad V0f)\ V5u)))))) \Rightarrow (p\ (ap\ (ap\ c\_2Ereal\_topology\_2Econtinuous\_on \\
& \quad V1g)\ V3t))))))
\end{aligned} \tag{48}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{49}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (50)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (51)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (52)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (53)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (54)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (57)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (58)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (59)$$



Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (60)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (62)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (63)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). (\forall V1s \in \\ & \quad (2^{ty\_2Erealax\_2Ereal}). (\forall V2t \in (2^{ty\_2Erealax\_2Ereal}). \\ & \quad ((p (ap (ap c\_2Ereal\_topology\_2Econtinuous\_on V0f) V1s)) \wedge \\ & \quad (((ap (ap (c\_2Epred\_set\_2EIMAGE ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\ & \quad V0f) V1s) = V2t) \wedge ((\forall V3x \in ty\_2Erealax\_2Ereal. (\forall V4y \in \\ & \quad ty\_2Erealax\_2Ereal. ((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\ & \quad V3x) V1s)) \wedge ((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V4y) \\ & \quad V1s)) \wedge ((ap V0f V3x) = (ap V0f V4y)))) \Rightarrow (V3x = V4y)))) \wedge (\forall V5u \in \\ & \quad (2^{ty\_2Erealax\_2Ereal}). ((p (ap (ap (c\_2Etopology\_2Eclosed\_in \\ & \quad ty\_2Erealax\_2Ereal) (ap (ap (c\_2Ereal\_topology\_2Esubtopology \\ & \quad ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Eeuclidean) V1s)) \\ & \quad V5u)) \Rightarrow (p (ap (ap (c\_2Etopology\_2Eclosed\_in ty\_2Erealax\_2Ereal) \\ & \quad (ap (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\ & \quad c\_2Ereal\_topology\_2Eeuclidean) V2t)) (ap (ap (c\_2Epred\_set\_2EIMAGE \\ & \quad ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) V0f) V5u)))))) \Rightarrow (\exists V6g \in \\ & \quad (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). (p (ap (ap c\_2Ereal\_topology\_2Ehomeomorphism \\ & \quad (ap (ap (c\_2Epair\_2E\_2C (2^{ty\_2Erealax\_2Ereal}) (2^{ty\_2Erealax\_2Ereal})) \\ & \quad V1s) V2t)) (ap (ap (c\_2Epair\_2E\_2C (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}) \\ & \quad (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})) V0f) V6g)))))) \end{aligned}$$