

thm_2Ereal__topology_2EINDEPENDENT__INJECTIVE__IMAGE (TMNMQ327oJAvERjbva5zigR64d9iNDMHBpn)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ecombin_2E_2S$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 4 We define $c_2Ecombin_2E_2K$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 5 We define $c_2Ecombin_2E_2I$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2E_2S A_27a (A_27a^{A_27a})) A_27a))$

Definition 6 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) P) P))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (2)$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS_prod A_27a A_27b) (ty_2Epair_2Eprod A_27a A_27b))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \quad (3)$$

Definition 11 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 12 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (4)$$

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \quad (5)$$

Let $c_2Erealx_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealx_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealx_2Ereal}) \quad (6)$$

Definition 13 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p\ (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge P\ x)$ of type $\iota \Rightarrow \iota$).

Definition 14 We define $c_2Erealx_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealx_2Ereal.(ap\ (c_2Emin_2E40\ (ty_2Erealx_2Ereal\ a)))$

Let $c_2Erealx_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealx_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (7)$$

Let $c_2Erealx_2Etrealeq : \iota$ be given. Assume the following.

$$c_2Erealx_2Etrealeq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (8)$$

Let $c_2Erealx_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealx_2Ereal_ABS_CLASS \in (ty_2Erealx_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (9)$$

Definition 15 We define $c_2Erealx_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 16 We define $c_2Erealx_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal.\lambda V1T2 \in ty_2Erealx_2Ereal$

Let $c_2Erealx_2Ereal_add : \iota$ be given. Assume the following.

$$c_2Erealx_2Ereal_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (10)$$

Definition 17 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 18 We define $c_2Ereal_topology_2Elinear$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})$

Definition 19 We define c_2Ebool_2E2F to be $(ap (c_2Ebool_2E_21) 2) (\lambda V0t \in 2.V0t)$.

Definition 20 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E2F)$.

Definition 21 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.V2t)))$

Definition 22 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.V2t))$

Definition 23 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21) 2) (\lambda V1t \in 2.V1t))$

Definition 24 We define $c_2Epred_set_2EEDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.V2t))$

Definition 25 We define $c_2Epred_set_2EDELETE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1x \in A_27a.(ap (ap c_2Ebool_2E_21) 2) (\lambda V2t \in 2.V2t))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \quad (11)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (12)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (13)$$

Definition 26 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (14)$$

Definition 27 We define $c_2Ereal_topology_2Esubspace$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (ap c_2Ebool_2E_21) 2) (\lambda V1t \in 2.V1t))$

Definition 28 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.V2t))$

Definition 29 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_2Epred_set_2ESUBSET) 2) (\lambda V1t \in 2.V1t))$

Definition 30 We define $c_2Etopology_2Ehull$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).\lambda V1s \in (2^{A_27a}).(ap (c_2Epred_set_2EBIGINTER) 2) (\lambda V2t \in 2.V2t))$

Definition 31 We define $c_2Ereal_topology_2Espan$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (ap c_2Etopology_2Ehull) 2) (\lambda V1t \in 2.V1t))$

Definition 32 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40) 2) (\lambda V1t \in 2.V1t))))$

Definition 33 We define $c_2Ereal_topology_2Edependent$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Ebool_2E_3F) 2) (\lambda V1t \in 2.V1t))$

Definition 34 We define $c_2Ereal_topology_2Eindependent$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap c_2Ebool_2E_3F) 2) (\lambda V1t \in 2.V1t))$

Definition 35 We define $c_2Eiterate_2Eneutral$ to be $\lambda A_27a : \iota.\lambda V0op \in ((A_27a^{A_27a})^{A_27a}).(ap (c_2Emin$

Definition 36 We define $c_2Eiterate_2Esupport$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0op \in ((A_27b^{A_27b})^{A_27b}).\lambda V$

Definition 37 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 38 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E_21 (2$

Definition 39 We define $c_2Eiterate_2EITSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in ((A_27a^{A_27a})^{A_27b}).\lambda V$

Definition 40 We define $c_2Eiterate_2Eiterate$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0op \in ((A_27b^{A_27b})^{A_27b}).\lambda V$

Definition 41 We define $c_2Eiterate_2ESum$ to be $\lambda A_27a : \iota.(ap (c_2Eiterate_2Eiterate A_27a ty_2Erealax$

Assume the following.

$$True \tag{15}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{16}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{17}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \tag{18}$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \tag{19}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \tag{21}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \tag{22}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\forall V1x \in A_27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A_27a.(\neg(p (ap V0P V2x))))) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\exists V1x \in A_27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A_27a.(\neg(p (ap V0P V2x))))) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).(((p V0P) \wedge (\forall V2x \in A_27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A_27a.((p V0P) \wedge (p (ap V1Q V3x))))) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).(((p V0P) \vee (\exists V2x \in A_27a.(p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in A_27a.((p V0P) \vee (p (ap V1Q V3x))))) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in 2.((\exists V2x \in A_27a.((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in A_27a.((p (ap V0P V3x)) \wedge (p V1Q))))) \quad (32)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p (ap V1P V2x)) \vee (p V0Q)))) \Leftrightarrow ((\forall V3x \in A.27a. (p (ap V1P V3x)) \vee (p V0Q)))))) \quad (33)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a. (p (ap V1Q V3x)))))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge (((\neg(p V0A) \vee (p V1B)) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B)))))) \quad (37)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (38)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x.27 \in 2. (\forall V2y \in 2. (\forall V3y.27 \in 2. (((((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \quad (39)$$

Assume the following.

$$(\forall V0y \in 2. (\forall V1x \in 2. (((p V0y) \Rightarrow (p V1x)) \Leftrightarrow ((\neg(p V1x)) \Rightarrow (\neg(p V0y)))))) \quad (40)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\forall V0P \in ((2^{A.27b})^{A.27a}). ((\forall V1x \in A.27a. (\exists V2y \in A.27b. (p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A.27b^{A.27a}). (\forall V4x \in A.27a. (p (ap (ap V0P V4x) (ap V3f V4x)))))) \quad (41)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ & 2. (((\exists V2x \in A_27a. (p\ (ap\ V0P\ V2x))) \Rightarrow (p\ V1Q))) \Leftrightarrow (\forall V3x \in \\ & A_27a. ((p\ (ap\ V0P\ V3x)) \Rightarrow (p\ V1Q)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27a^{A_27c}). \\ & (\forall V2x \in A_27c. ((ap\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27c\ A_27b\ A_27a) \\ & V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \end{aligned} \quad (43)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Ecombin_2EI \\ A_27a)\ V0x) = V0x)) \quad (44)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (ty_2Erealax_2Ereal^{A_27b}). \\ & (\forall V2s \in (2^{A_27a}). ((\forall V3x \in A_27a. (\forall V4y \in A_27a. \\ & (((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V2s)) \wedge ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V4y)\ V2s)) \wedge ((ap\ V0f\ V3x) = (ap\ V0f\ V4y)))))) \Rightarrow (V3x = V4y)))))) \Rightarrow \\ & ((ap\ (ap\ (c_2Eiterate_2ESum\ A_27b)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\ & A_27a\ A_27b)\ V0f)\ V2s))\ V1g) = (ap\ (ap\ (c_2Eiterate_2ESum\ A_27a) \\ & V2s)\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27a\ ty_2Erealax_2Ereal\ A_27b)\ V1g) \\ & V0f)))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0P \in (2^{A_27a}). (\forall V1f \in (A_27a^{A_27b}). (\forall V2s \in \\ & (2^{A_27b}). ((\exists V3y \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\ & V3y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_27b\ A_27a)\ V1f)\ V2s)))) \wedge (\\ & p\ (ap\ V0P\ V3y)))) \Leftrightarrow (\exists V4x \in A_27b. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27b)\ V4x)\ V2s)) \wedge (p\ (ap\ V0P\ (ap\ V1f\ V4x)))))))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in (A_27b^{A_27a}). (\forall V1s \in (2^{A_27a}). (\forall V2t \in \\ & (2^{A_27b}). (((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27b)\ V2t)) \wedge (p\ (\\ & ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27b)\ V2t)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\ & A_27a\ A_27b)\ V0f)\ V1s)))) \Leftrightarrow (\exists V3s_27 \in (2^{A_27a}). ((p\ (ap\ (\\ & c_2Epred_set_2EFINITE\ A_27a)\ V3s_27)) \wedge ((p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET \\ & A_27a)\ V3s_27)\ V1s)) \wedge (V2t = (ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_27a \\ & A_27b)\ V0f)\ V3s_27)))))))))) \end{aligned} \quad (47)$$

Assume the following.

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).((p (ap\ c_2Ereal_topology_2Elinear\ V0f)) \Rightarrow ((ap\ V0f\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) = (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)))) \quad (48)$$

Assume the following.

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1c \in ty_2Erealax_2Ereal.(\forall V2x \in ty_2Erealax_2Ereal.((p (ap\ c_2Ereal_topology_2Elinear\ V0f)) \Rightarrow ((ap\ V0f\ (ap\ (ap\ c_2Erealax_2Ereal_mul\ V1c)\ V2x)) = (ap\ (ap\ c_2Erealax_2Ereal_mul\ V1c)\ (ap\ V0f\ V2x))))))) \quad (49)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1g \in (ty_2Erealax_2Ereal^{A.27a}).(\forall V2s \in (2^{A.27a}).(((p (ap\ c_2Ereal_topology_2Elinear\ V0f)) \wedge (p (ap (c_2Epred_set_2EFINITE\ A.27a)\ V2s))) \Rightarrow ((ap\ V0f\ (ap (ap (c_2Eiterate_2ESum\ A.27a)\ V2s)\ V1g)) = (ap (ap (c_2Eiterate_2ESum\ A.27a)\ V2s)\ (ap (ap (c_2Ecombin_2Eo\ A.27a)\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ V0f)\ V1g)))))) \quad (50)$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1x \in ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ V1x)\ V0s)) \Rightarrow (p (ap (ap (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ V1x)\ (ap\ c_2Ereal_topology_2Espan\ V0s)))))) \quad (51)$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}).(p (ap (ap (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) (ap\ c_2Ereal_topology_2Espan\ V0s)))) \quad (52)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1c \in ty_2Erealax_2Ereal.(\forall V2s \in (2^{ty_2Erealax_2Ereal}).((p (ap (ap (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ V0x)\ (ap\ c_2Ereal_topology_2Espan\ V2s))) \Rightarrow (p (ap (ap (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ (ap (ap\ c_2Erealax_2Ereal_mul\ V1c)\ V0x)) (ap\ c_2Ereal_topology_2Espan\ V2s)))))) \quad (53)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{ty_2Erealx_2Ereal}). \\
& \quad (\forall V1f \in (ty_2Erealx_2Ereal^{A.27a}). (\forall V2t \in (2^{A.27a}). \\
& \quad ((p\ (ap\ (c_2Epred_set_2EFINITE\ A.27a)\ V2t)) \wedge (\forall V3x \in A.27a. \\
& \quad ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V3x)\ V2t)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad ty_2Erealx_2Ereal)\ (ap\ V1f\ V3x))\ (ap\ c_2Ereal_topology_2Espan \\
& \quad V0s)))))) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealx_2Ereal)\ (ap\ (\\
& \quad ap\ (c_2Eiterate_2ESum\ A.27a)\ V2t)\ V1f))\ (ap\ c_2Ereal_topology_2Espan \\
& \quad V0s))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (2^{ty_2Erealx_2Ereal}). ((p\ (ap\ c_2Ereal_topology_2Edependent \\
& V0p)) \Leftrightarrow (\exists V1s \in (2^{ty_2Erealx_2Ereal}). (\exists V2u \in (ty_2Erealx_2Ereal^{ty_2Erealx_2Ereal}). \\
& \quad ((p\ (ap\ (c_2Epred_set_2EFINITE\ ty_2Erealx_2Ereal)\ V1s)) \wedge (\\
& \quad (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ ty_2Erealx_2Ereal)\ V1s)\ V0p)) \wedge \\
& \quad ((\exists V3v \in ty_2Erealx_2Ereal. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& ty_2Erealx_2Ereal)\ V3v)\ V1s)) \wedge (\neg((ap\ V2u\ V3v) = (ap\ c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))))) \wedge ((ap\ (ap\ (c_2Eiterate_2ESum\ ty_2Erealx_2Ereal \\
& V1s)\ (\lambda V4v \in ty_2Erealx_2Ereal. (ap\ (ap\ c_2Erealx_2Ereal_mul \\
& (ap\ V2u\ V4v))\ V4v))) = (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)))))))))
\end{aligned} \tag{55}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{56}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))
\end{aligned} \tag{59}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee (\neg(\\
& \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{61}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \quad (62)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (63)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (\neg(p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (64)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (65)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (66)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (67)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (68)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (69)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (70)$$

Theorem 1

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V1s \in (2^{ty_2Erealax_2Ereal}). (((p (ap\ c_2Ereal_topology_2EIndependent\ V1s)) \wedge ((p (ap\ c_2Ereal_topology_2Elinear\ V0f)) \wedge (\forall V2x \in ty_2Erealax_2Ereal. (\forall V3y \in ty_2Erealax_2Ereal. (((p (ap (ap (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ V2x) (ap\ c_2Ereal_topology_2Espan\ V1s))) \wedge ((p (ap (ap (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ V3y) (ap\ c_2Ereal_topology_2Espan\ V1s))) \wedge ((ap\ V0f\ V2x) = (ap\ V0f\ V3y)))))) \Rightarrow (p (ap\ c_2Ereal_topology_2EIndependent\ (ap (ap (c_2Epred_set_2EIMAGE\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ V0f)\ V1s)))))))))$$