

thm_2Ereal__topology_2EINJECTIVE__IMP__ISOMETRIC (TMLp2XVmMmyTmZXd3idgBCPGiM9ABnn4deJ)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ecombin_2EK$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.(\lambda V0x \in A.\lambda V1y \in A.\lambda V0x)$

Definition 3 We define $c_2Ecombin_2ES$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda A.\lambda c : \iota.(\lambda V0f \in ((A.\lambda c^{A.\lambda b})^{A.\lambda a}))$

Definition 4 We define $c_2Ecombin_2EI$ to be $\lambda A.\lambda a : \iota.(ap (ap (c_2Ecombin_2ES A.\lambda a (A.\lambda a^{A.\lambda a})) A.\lambda a))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \tag{3}$$

Definition 5 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A.\lambda a})).(ap (ap (c_2Emin_2E_3D (2^{A.\lambda a})) (\lambda V1t \in 2.V1t)) (\lambda V2t \in 2.V2t))$

Definition 7 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (5)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (6)$$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num$

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x. x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 13 We define $c_2Ebool_2E_3F$ to be $\lambda A. 2^A : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 14 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 15 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 16 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 17 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (7)$$

Definition 18 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 19 We define c_2Ebool_2ECOND to be $\lambda A. 2^A : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A. 2^A. (\lambda V2t2 \in A. 2^A. ($

Definition 20 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ (ap\ (ap\ (c_2Ebool_2E$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Definition 21 We define $c_2Enumeral_2EiiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ c_2Enum_2ESUC\ (ap$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Definition 22 We define $c_2Earithmic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmic_2EBIT2))$

Definition 23 We define $c_2Enumeral_2EiDUB$ to be $\lambda V0x \in ty_2Enum_2Enum.(ap (ap c_2Earithmic_2EBIT2))$

Definition 24 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (11)$$

Definition 25 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 26 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}).(ap (c_2Ebool_2EIN A_27a s t))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (12)$$

Definition 27 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b.(ap (c_2Ebool_2EIN A_27a x y))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (13)$$

Definition 28 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}).(ap (c_2Ebool_2EIN A_27a x s))$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty ty_2Ehreal_2Ehreal \quad (14)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty ty_2Erealax_2Ereal \quad (15)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (16)$$

Definition 29 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E40 V0a))$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)}) \quad (17)$$

Definition 30 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 31 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 32 We define $c_2Ereal_2Ereal_ge$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Let $c_2Erealax_2Etrealm_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) \quad (18)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (19)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (20)$$

Definition 33 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 34 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS)$

Definition 35 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 36 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E1))$

Definition 37 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) \quad (21)$$

Definition 38 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_inv)$

Let $c_2Erealax_2Etrealm_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal) \quad (22)$$

Definition 39 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 40 We define c_2Ereal_2E2F to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 41 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (23)$$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (24)$$

Definition 42 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 43 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (25)$$

Definition 44 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECONI$

Definition 45 We define $c_2Ereal_topology_2Esubspace$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (ap\ c_2Ebool$

Let $c_2Ereal_topology_2ESphere : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2ESphere \in ((2^{ty_2Erealax_2Ereal})^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (26)$$

Definition 46 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2$

Let $c_2Ereal_topology_2Ecball : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2Ecball \in ((2^{ty_2Erealax_2Ereal})^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (27)$$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (28)$$

Definition 47 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (c_2Ebool_2E2$

Definition 48 We define $c_2Ereal_topology_2Einterior$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (c_2Epred_set$

Definition 49 We define $c_2Ereal_topology_2Elimit_point_of$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1s \in ($

Definition 50 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2$

Definition 51 We define $c_2Ereal_topology_2Eclosure$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (ap\ (c_2Epred$

Definition 52 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2$

Definition 53 We define $c_2Ereal_topology_2Efrontier$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (ap\ (c_2Epred$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) V1n) = c_2Enum_2E0) \Leftrightarrow ((V0m = c_2Enum_2E0) \wedge (V1n = c_2Enum_2E0)))) \quad (34)$$

Assume the following.

$$((\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D V0n) c_2Enum_2E0)) \Leftrightarrow (V0n = c_2Enum_2E0))) \wedge (\forall V1m \in ty_2Enum_2Enum. (\forall V2n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D V1m) (ap c_2Enum_2ESUC V2n))) \Leftrightarrow ((V1m = (ap c_2Enum_2ESUC V2n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D V1m) V2n))))))) \quad (35)$$

Assume the following.

$$True \quad (36)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (37)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg (p V0t)))) \quad (39)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (40)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (41)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow (\neg (p V0t)))) \quad (42)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg (p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (43)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))) \quad (44)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg (p \ V0t))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{47}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \tag{48}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{49}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\
& A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg (p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p \ V0t))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in \\
& A_27a.(((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2ET) \ V0t1) \\
& V1t2) = V0t1) \wedge ((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2EF) \\
& V0t1) \ V1t2) = V1t2))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\forall V1x \in A_27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\exists V2x \in A_27a.(\neg(p \ (ap \ V0P \ V2x)))))) \tag{53}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ 2^{A.27a}). ((p \ V0P) \wedge (\forall V2x \in A.27a. (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow (\forall V3x \in \\ A.27a. ((p \ V0P) \wedge (p \ (ap \ V1Q \ V3x)))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ 2^{A.27a}). ((p \ V0P) \vee (\exists V2x \in A.27a. (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow (\exists V3x \in \\ A.27a. ((p \ V0P) \vee (p \ (ap \ V1Q \ V3x)))))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in \\ 2. ((\exists V2x \in A.27a. ((p \ (ap \ V0P \ V2x)) \wedge (p \ V1Q))) \Leftrightarrow ((\exists V3x \in \\ A.27a. (p \ (ap \ V0P \ V3x)) \wedge (p \ V1Q)))))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ 2^{A.27a}). ((\forall V2x \in A.27a. ((p \ V0P) \vee (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((p \\ V0P) \vee (\forall V3x \in A.27a. (p \ (ap \ V1Q \ V3x)))))) \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. ((\neg((p \ V0A) \Rightarrow (p \ V1B))) \Leftrightarrow ((p \ V0A) \wedge \\ (\neg(p \ V1B)))))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p \ V0A) \vee (\\ (p \ V1B) \vee (p \ V2C))) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C)))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (((p \ V0A) \vee (p \ V1B)) \Leftrightarrow ((p \ V1B) \vee \\ (p \ V0A)))))) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg(\\ p \ V0A) \vee (\neg(p \ V1B)))) \wedge ((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A) \wedge (\neg(p \ V1B)))))))))) \end{aligned} \quad (61)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p \ V0t1) \Rightarrow \\ ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \end{aligned} \quad (62)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2. (\forall V1x.27 \in 2. (\forall V2y \in 2. (\forall V3y.27 \in \\ 2. (((p \ V0x) \Leftrightarrow (p \ V1x.27)) \wedge ((p \ V1x.27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y.27)))) \Rightarrow \\ ((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x.27) \Rightarrow (p \ V3y.27)))))) \end{aligned} \quad (63)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A-27a}). (\forall V1a \in \\ A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p (ap\ V0P\ V2x)))) \Leftrightarrow (p (\\ ap\ V0P\ V1a)))))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0P \in ((2^{A-27b})^{A-27a}). ((\forall V1x \in A_27a. (\exists V2y \in \\ A_27b. (p (ap (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A_27b^{A-27a}). (\\ \forall V4x \in A_27a. (p (ap (ap\ V0P\ V4x)\ (ap\ V3f\ V4x))))))) \end{aligned} \quad (65)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap (c.2Ecombin_2El \\ A_27a)\ V0x) = V0x)) \end{aligned} \quad (66)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (\\
& ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (\\
& ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (\\
& ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap \\
& c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC \\
& (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ c_2Earithmetic_2EZERO)) = (\\
& ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (\\
& ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT1 \\
& V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (((ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Earithmetic_2EBIT1\ V0n)) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiDUB\ V0n))) \wedge \\
& \quad (((ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Earithmetic_2EBIT2\ V0n)) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Earithmetic_2EBIT1\ V0n))) \wedge ((ap\ c_2Enumeral_2EiDUB\ c_2Earithmetic_2EZERO) = c_2Earithmetic_2EZERO)))) \\
& \hspace{15em} (69)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. ((ap\ (ap\ c_2Earithmetic_2E_2A\ c_2Earithmetic_2EZERO)\ V0n) = c_2Earithmetic_2EZERO) \wedge \\
& \quad (((ap\ (ap\ c_2Earithmetic_2E_2A\ V0n)\ c_2Earithmetic_2EZERO) = c_2Earithmetic_2EZERO) \wedge (((ap\ (ap\ c_2Earithmetic_2E_2A\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ V1m) = (ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Enumeral_2EiDUB\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0n)\ V1m))))\ V1m))) \wedge ((ap\ (ap\ c_2Earithmetic_2E_2A\ (ap\ c_2Earithmetic_2EBIT2\ V0n))\ V1m) = (ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0n)\ V1m))))\ V1m))))))))) \\
& \hspace{15em} (70)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b))) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \\
& \hspace{15em} (71)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t))))))) \\
& \hspace{15em} (72)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}). (\forall V1v \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC\ A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \\
& \hspace{15em} (73)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). ((\exists V1x \in A_27a. (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1x)\ V0s))) \Leftrightarrow (\neg (V0s = (c_2Epred_set_2EEMPTY\ A_27a)))))) \\
& \hspace{15em} (74)
\end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\ & V2x)\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (75) \\ & (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V2x)\ V1t)))))) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ & A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (ap\ (ap\ (c_2Epred_set_2EINSERT \\ & A_27a)\ V1y)\ (c_2Epred_set_2EEMPTY\ A_27a)))) \Leftrightarrow (V0x = V1y))) \end{aligned} \quad (76)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b^{A_27a}). \\ & ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\ & A_27a\ A_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\ & (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s)))))) \end{aligned} \quad (77)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0P \in (2^{A_27a}). (\forall V1f \in (A_27a^{A_27b}). (\forall V2s \in \\ & (2^{A_27b}). ((\forall V3y \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\ & V3y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_27b\ A_27a)\ V1f)\ V2s))) \Rightarrow (78) \\ & p\ (ap\ V0P\ V3y)))) \Leftrightarrow (\forall V4x \in A_27b. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27b)\ V4x)\ V2s)) \Rightarrow (p\ (ap\ V0P\ (ap\ V1f\ V4x)))))) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0P \in (2^{A_27a}). (\forall V1f \in (A_27a^{A_27b}). (\forall V2s \in \\ & (2^{A_27b}). ((\exists V3y \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\ & V3y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_27b\ A_27a)\ V1f)\ V2s))) \wedge (79) \\ & p\ (ap\ V0P\ V3y)))) \Leftrightarrow (\exists V4x \in A_27b. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27b)\ V4x)\ V2s)) \wedge (p\ (ap\ V0P\ (ap\ V1f\ V4x)))))) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealx_2Ereal. (\forall V1y \in ty_2Erealx_2Ereal. \\ & ((ap\ (ap\ c_2Erealx_2Ereal_add\ V0x)\ V1y) = (ap\ (ap\ c_2Erealx_2Ereal_add \\ & V1y)\ V0x)))) \end{aligned} \quad (80)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
V0x) (ap (ap c_2Erealax_2Ereal_add V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_add \\
& (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) V2z))))))
\end{aligned} \tag{81}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = V0x))
\end{aligned} \tag{82}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& (ap c_2Erealax_2Ereal_neg V0x)) V0x) = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))
\end{aligned} \tag{83}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. (((p (ap (ap c_2Erealax_2Ereal_lt \\
V0x) V1y)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt V1y) V2z))) \Rightarrow (p (ap \\
& (ap c_2Erealax_2Ereal_lt V0x) V2z))))))
\end{aligned} \tag{84}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_mul V0x) V1y) = (ap (ap c_2Erealax_2Ereal_mul \\
& V1y) V0x))))
\end{aligned} \tag{85}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
V0x) (ap (ap c_2Erealax_2Ereal_mul V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_mul \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)) V2z))))))
\end{aligned} \tag{86}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (\\
ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) V0x) = V0x))
\end{aligned} \tag{87}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
c_2Enum_2E0)) V0x)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
c_2Enum_2E0)) V1y))) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
c_2Enum_2E0)) (ap (ap c_2Erealax_2Ereal_mul V0x) V1y))))))
\end{aligned} \tag{88}$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_add V0x) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = V0x)) \quad (89)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_add V0x) (ap c_2Erealax_2Ereal_neg V0x)) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \quad (90)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_mul (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \quad (91)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_mul V0x) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \quad (92)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. (\forall V2z \in ty_2Erealax_2Ereal.((p (ap (ap c_2Erealax_2Ereal_lt (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) (ap (ap c_2Erealax_2Ereal_add V0x) V2z)))) \Leftrightarrow (p (ap (ap c_2Erealax_2Ereal_lt V1y) V2z)))))) \quad (93)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. ((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \vee (p (ap (ap c_2Ereal_2Ereal_lte V1y) V0x)))))) \quad (94)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(p (ap (ap c_2Ereal_2Ereal_lte V0x) V0x))) \quad (95)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. (\forall V2z \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Erealax_2Ereal_lt V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V1y) V2z)))) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt V0x) V2z)))))) \quad (96)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. (((p (ap (ap c_2Ereal_2Ereal_lte \\
V0x) V1y)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt V1y) V2z))) \Rightarrow (p (ap \\
& (ap c_2Erealax_2Ereal_lt V0x) V2z))))))
\end{aligned} \tag{97}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. (((p (ap (ap c_2Ereal_2Ereal_lte \\
V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V1y) V2z))) \Rightarrow (p (ap (\\
& ap c_2Ereal_2Ereal_lte V0x) V2z))))))
\end{aligned} \tag{98}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte \\
V1y) V0x))) \Leftrightarrow (V0x = V1y)))
\end{aligned} \tag{99}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
c_2Enum_2E0)) V0x)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
c_2Enum_2E0)) V1y))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
c_2Enum_2E0)) (ap (ap c_2Erealax_2Ereal_mul V0x) V1y))))))
\end{aligned} \tag{100}$$

Assume the following.

$$\begin{aligned}
& (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
c_2Enum_2E0)) (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmic_2ENUMERAL \\
(ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO))))))
\end{aligned} \tag{101}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (ap (ap c_2Erealax_2Ereal_add (ap c_2Ereal_2Ereal_of_num \\
V0m)) (ap c_2Ereal_2Ereal_of_num V1n)) = (ap c_2Ereal_2Ereal_of_num \\
& (ap (ap c_2Earithmic_2E_2B V0m) V1n))))
\end{aligned} \tag{102}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((\neg (V1y = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \Rightarrow ((ap (\\
ap c_2Erealax_2Ereal_mul (ap (ap c_2Ereal_2E_2F V0x) V1y)) V1y) = \\
& V0x))))
\end{aligned} \tag{103}$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Ereal_2Ereal_sub (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = (ap c_2Erealax_2Ereal_neg V0x))) \quad (104)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V1y))) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) (ap (ap c_2Ereal_2E_2F V0x) V1y)))))) \quad (105)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(((ap c_2Ereal_2Eabs V0x) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \Leftrightarrow (V0x = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)))) \quad (106)$$

Assume the following.

$$((ap c_2Ereal_2Eabs (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \quad (107)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap c_2Ereal_2Eabs (ap c_2Erealax_2Ereal_neg V0x)) = (ap c_2Ereal_2Eabs V0x))) \quad (108)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.(((ap c_2Ereal_2Eabs (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)) = (ap (ap c_2Erealax_2Ereal_mul (ap c_2Ereal_2Eabs V0x)) (ap c_2Ereal_2Eabs V1y)))))) \quad (109)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((\neg (V0x = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \Leftrightarrow (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) (ap c_2Ereal_2Eabs V0x)))))) \quad (110)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap c_2Ereal_2Eabs (ap c_2Ereal_2Eabs V0x)) = (ap c_2Ereal_2Eabs V0x))) \quad (111)$$

Assume the following.

$$\begin{aligned}
& (\forall V0y \in ty_2Erealax_2Ereal. ((\neg(V0y = (ap\ c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0))) \Rightarrow (\forall V1x \in ty_2Erealax_2Ereal. ((ap\ c_2Ereal_2Eabs \\
& (ap\ (ap\ c_2Ereal_2E_2F\ V1x)\ V0y)) = (ap\ (ap\ c_2Ereal_2E_2F\ (ap\ c_2Ereal_2Eabs \\
& \quad V1x))\ (ap\ c_2Ereal_2Eabs\ V0y))))))
\end{aligned} \tag{112}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap\ (ap\ c_2Erealax_2Ereal_mul\ V0x)\ (ap\ c_2Erealax_2Ereal_neg \\
& V1y)) = (ap\ c_2Erealax_2Ereal_neg\ (ap\ (ap\ c_2Erealax_2Ereal_mul \\
& \quad V0x)\ V1y))))))
\end{aligned} \tag{113}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap\ (ap\ c_2Erealax_2Ereal_mul\ (ap\ c_2Erealax_2Ereal_neg\ V0x)) \\
& V1y) = (ap\ c_2Erealax_2Ereal_neg\ (ap\ (ap\ c_2Erealax_2Ereal_mul \\
& \quad V0x)\ V1y))))))
\end{aligned} \tag{114}$$

Assume the following.

$$\begin{aligned}
& (\forall V0y \in ty_2Erealax_2Ereal. (\forall V1x \in ty_2Erealax_2Ereal. \\
& ((p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ V1x)\ V0y)) \Leftrightarrow (\neg(p\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\
& \quad V0y)\ V1x))))))
\end{aligned} \tag{115}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((p\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\
& V1y)\ V2z)) \Rightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ (ap\ c_2Erealax_2Ereal_add \\
& \quad V0x)\ V1y))\ (ap\ (ap\ c_2Erealax_2Ereal_add\ V0x)\ V2z))))))
\end{aligned} \tag{116}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ c_2Erealax_2Ereal_neg\ V0x)) \\
& V1y)) \Leftrightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0))\ (ap\ (ap\ c_2Erealax_2Ereal_add\ V0x)\ V1y))))))
\end{aligned} \tag{117}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ c_2Erealax_2Ereal_neg\ V0x)) \\
& (ap\ c_2Erealax_2Ereal_neg\ V1y)) \Leftrightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\
& \quad V1y)\ V0x))))))
\end{aligned} \tag{118}$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap\ c_2Erealax_2Ereal_neg\ (ap\ c_2Erealax_2Ereal_neg\ V0x)) = V0x)) \quad (119)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.((p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ V0x)\ (ap\ c_2Erealax_2Ereal_neg\ V1y))) \Leftrightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ (ap\ c_2Erealax_2Ereal_add\ V0x)\ V1y))\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)))))) \quad (120)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.(\forall V2z \in ty_2Erealax_2Ereal.((p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))\ V2z)) \Rightarrow ((p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ V0x)\ (ap\ (ap\ c_2Ereal_2E_2F\ V1y)\ V2z))) \Leftrightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ (ap\ c_2Erealax_2Ereal_mul\ V0x)\ V2z))\ V1y)))))) \quad (121)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.(\forall V2z \in ty_2Erealax_2Ereal.((p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))\ V2z)) \Rightarrow ((p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ (ap\ c_2Ereal_2E_2F\ V0x)\ V2z))\ V1y)) \Leftrightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ V0x)\ (ap\ (ap\ c_2Erealax_2Ereal_mul\ V1y)\ V2z)))))) \quad (122)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.(\forall V2z \in ty_2Erealax_2Ereal.((ap\ (ap\ c_2Erealax_2Ereal_mul\ (ap\ (ap\ c_2Erealax_2Ereal_add\ V0x)\ V1y))\ V2z) = (ap\ (ap\ c_2Erealax_2Ereal_add\ (ap\ (ap\ c_2Erealax_2Ereal_mul\ V0x)\ V2z))\ (ap\ (ap\ c_2Erealax_2Ereal_mul\ V1y)\ V2z)))))) \quad (123)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ c_2Ereal_2Ereal_of_num\ V0m))\ (ap\ c_2Ereal_2Ereal_of_num\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V0m)\ V1n)))) \quad (124)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap\ (ap\ c_2Erealax_2Ereal_mul\ (ap\ c_2Ereal_2Ereal_of_num\ V0m))\ (ap\ c_2Ereal_2Ereal_of_num\ V1n)) = (ap\ c_2Ereal_2Ereal_of_num\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ V1n)))) \quad (125)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap\ c_2Ereal_topology_2EDist\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal)\ V0x)\ V1y))) = (ap\ c_2Ereal_2Eabs\ (ap\ (ap\ c_2Ereal_2Ereal_sub \\
& V0x)\ V1y))))))
\end{aligned} \tag{126}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). ((p\ (\\
& ap\ c_2Ereal_topology_2Elinear\ V0f))) \Rightarrow ((ap\ V0f\ (ap\ c_2Ereal_of_num \\
& c_2Enum_2E0)) = (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))))
\end{aligned} \tag{127}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V1c \in \\
& ty_2Erealax_2Ereal. (\forall V2x \in ty_2Erealax_2Ereal. ((p\ (ap \\
& c_2Ereal_topology_2Elinear\ V0f))) \Rightarrow ((ap\ V0f\ (ap\ (ap\ c_2Erealax_2Ereal_mul \\
& V1c)\ V2x))) = (ap\ (ap\ c_2Erealax_2Ereal_mul\ V1c)\ (ap\ V0f\ V2x))))))
\end{aligned} \tag{128}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1e \in ty_2Erealax_2Ereal. \\
& ((ap\ c_2Ereal_topology_2Esphere\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal)\ V0x)\ V1e))) = (ap\ (c_2Epred_set_2EGSPEC\ ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal)\ (\lambda V2y \in ty_2Erealax_2Ereal. (ap\ (ap\ (c_2Epair_2E_2C \\
& ty_2Erealax_2Ereal\ 2)\ V2y)\ (ap\ (ap\ (c_2Emin_2E_3D\ ty_2Erealax_2Ereal) \\
& (ap\ c_2Ereal_topology_2EDist\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal)\ V0x)\ V2y))))\ V1e))))))
\end{aligned} \tag{129}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Erealax_2Ereal. (\forall V1e \in ty_2Erealax_2Ereal. \\
& ((ap\ c_2Ereal_topology_2Efrontier\ (ap\ c_2Ereal_topology_2Ecball \\
& (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal) \\
& V0a)\ V1e)))) = (ap\ c_2Ereal_topology_2Esphere\ (ap\ (ap\ (c_2Epair_2E_2C \\
& ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ V0a)\ V1e))))))
\end{aligned} \tag{130}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}). ((p\ (ap\ c_2Ereal_topology_2Ecompact \\
& V0s))) \Rightarrow (p\ (ap\ c_2Ereal_topology_2EClosed\ V0s))))
\end{aligned} \tag{131}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}). (\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& (((p\ (ap\ c_2Ereal_topology_2EClosed\ V0s)) \wedge (p\ (ap\ c_2Ereal_topology_2Ecompact \\
& V1t)))) \Rightarrow (p\ (ap\ c_2Ereal_topology_2Ecompact\ (ap\ (ap\ (c_2Epred_set_2EINTER \\
& ty_2Erealax_2Ereal)\ V0s)\ V1t))))))
\end{aligned} \tag{132}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1e \in ty_2Erealax_2Ereal. \\
& (p (ap c_2Ereal_topology_2Ecompact (ap c_2Ereal_topology_2Ecball \\
& (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V0x) V1e))))))
\end{aligned} \tag{133}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}). ((p (ap c_2Ereal_topology_2Ecompact \\
& V0s)) \Rightarrow (p (ap c_2Ereal_topology_2Ecompact (ap c_2Ereal_topology_2Efrontier \\
& V0s))))))
\end{aligned} \tag{134}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V1s \in \\
& (2^{ty_2Erealax_2Ereal}). ((p (ap c_2Ereal_topology_2Elinear \\
& V0f)) \Rightarrow (p (ap (ap c_2Ereal_topology_2Econtinuous_on V0f) V1s))))))
\end{aligned} \tag{135}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V1s \in \\
& (2^{ty_2Erealax_2Ereal}). (((p (ap (ap c_2Ereal_topology_2Econtinuous_on \\
& V0f) V1s)) \wedge (p (ap c_2Ereal_topology_2Ecompact V1s))) \Rightarrow (p (ap \\
& c_2Ereal_topology_2Ecompact (ap (ap (c_2Epred_set_2EIMAGE \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0f) V1s))))))
\end{aligned} \tag{136}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}). (\forall V1a \in ty_2Erealax_2Ereal. \\
& (((p (ap c_2Ereal_topology_2EClosed V0s)) \wedge (\neg(V0s = (c_2Epred_set_2EEMPTY \\
& ty_2Erealax_2Ereal)))) \Rightarrow (\exists V2x \in ty_2Erealax_2Ereal. (\\
& (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V2x) V0s)) \wedge (\forall V3y \in \\
& ty_2Erealax_2Ereal. ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V3y) V0s)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_topology_2EDist \\
& (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V1a) V2x))) (ap c_2Ereal_topology_2EDist (ap (ap (c_2Epair_2E_2C \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V1a) V3y))))))))))
\end{aligned} \tag{137}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{138}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{139}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{140}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (141)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (142)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (143)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (144)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (145)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (146)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (147)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (148)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (149)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (150)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (151)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (152)$$

Theorem 1

$$\begin{aligned} & (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in \\ & (2^{ty_2Erealax_2Ereal}).(((p (ap c_2Ereal_topology_2EClosed \\ V1s)) \wedge ((p (ap c_2Ereal_topology_2Esubspace V1s)) \wedge ((p (ap c_2Ereal_topology_2Elinear \\ V0f)) \wedge (\forall V2x \in ty_2Erealax_2Ereal.(((p (ap (ap (c_2Ebool_2EIN \\ ty_2Erealax_2Ereal) V2x) V1s)) \wedge ((ap V0f V2x) = (ap c_2Ereal_2Ereal_of_num \\ c_2Enum_2E0)))) \Rightarrow (V2x = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)))))) \Rightarrow \\ & (\exists V3e \in ty_2Erealax_2Ereal.((p (ap (ap c_2Erealax_2Ereal_lt \\ (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V3e)) \wedge (\forall V4x \in \\ ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\ V4x) V1s)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_ge (ap c_2Ereal_2Eabs (\\ ap V0f V4x))) (ap (ap c_2Erealax_2Ereal_mul V3e) (ap c_2Ereal_2Eabs \\ V4x)))))))))))))) \end{aligned}$$