thm_2Ereal__topology_2EINJECTIVE__MAP__OPEN__IFF__CLOS (TMUtzSRZ2WyqaJDheZrwrNnBsrSwJb1VarN)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2EF$

Let $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \tag{1}$$

Let $c\_2Etopology\_2Eopen\_\_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Eopen\_\_in\ A\_27a \in ((2^{(2^{A\_27a})})^{(ty\_2Etopology\_2Etopology\ A\_27a)}) \tag{2}$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Definition 7 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

Let $ty\_2Epair\_2Eprod : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}\forall A1.nonempty\ A1{\Rightarrow}nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (4)$$

Let $c\_2Epair\_2EABS\_\_prod : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (5)$$

Definition 9 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Let $c\_2Ereal\_\_topology\_2EDist : \iota$ be given. Assume the following.

$$c\_2Ereal\_\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \quad (6)$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (7)$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \quad (8)$$

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A\wedge p$ *of type* $\iota{\Rightarrow}\iota$.

Definition 11 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(t$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehr} \quad (9)$$

Definition 12 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \quad (10)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (11)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (12)$$

Definition 13 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (13)$$

Definition 14 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 15 *We define* c_2Ereal__topology_2EOpen *to be* $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap$ (c_2Ebool_2E_2

Let $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})}) \quad (14)$$

Definition 16 *We define* c_2Ereal__topology_2Eeuclidean *to be* $(ap\ (c\_2Etopology\_2Etopology\ ty\_2Erealax$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \quad (15)$$

Definition 17 *We define* c_2Epred__set_2EINTER *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_$

Definition 18 *We define* c_2Ereal__topology_2Esubtopology *to be* $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Eto$

Definition 19 *We define* c_2Epred__set_2EBIGUNION *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_\_s$

Definition 20 *We define* c_2Etopology_2Etopspace *to be* $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology$

Definition 21 *We define* c_2Epred__set_2EDIFF *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2$

Definition 22 *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ ($

Definition 23 *We define* c_2Etopology_2Eclosed__in *to be* $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology$

Let $c\_2Ereal\_\_topology\_2Ehomeomorphism : \iota$ be given. Assume the following.

$$c\_2Ereal\_\_topology\_2Ehomeomorphism \in ((2^{(ty\_2Epair\_2Eprod\ (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})\ (ty\_2Ereal} \quad (16)$$

Definition 24 *We define* c_2Epred__set_2EIMAGE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in$

Definition 25 *We define* c_2Ereal__topology_2Econtinuous__on *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Ereal}$

Assume the following.

$$True \quad (17)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(\forall V2z \in A\_27a.(((V0x = V1y) \wedge (V1y = V2z)) \Rightarrow (V0x = V2z))))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{23}$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V2t \in (2^{ty\_2Erealax\_2Ereal}).(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econtinuous\_\_on}\ V0f)\ V1s)) \wedge (((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V0f)\ V1s) = V2t) \wedge (\forall V3x \in ty\_2Erealax\_2Ereal.(\forall V4y \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V3x)\ V1s)) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V4y)\ V1s)) \wedge ((ap\ V0f\ V3x) = (ap\ V0f\ V4y)))) \Rightarrow (V3x = V4y)))))) \Rightarrow ((\exists V5g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(p\ (ap\ (ap\ c\_2Ereal\_\_topology\_2Ehomeomorphism\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal}))\ V1s)\ V2t))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})\ (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}))\ V0f)\ V5g)))) \Leftrightarrow (\forall V6u \in (2^{ty\_2Erealax\_2Ereal}).((p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean})\ V1s))\ V6u)) \Rightarrow (p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean})\ V2t))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V0f)\ V6u)))))))))) \tag{24}$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V2t \in (2^{ty\_2Erealax\_2Ereal}). (((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econtinuous\_\_on}\ V0f)\ V1s))\wedge (((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V0f)\ V1s) = V2t)\wedge(\forall V3x \in ty\_2Erealax\_2Ereal.(\forall V4y \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V3x)\ V1s))\wedge((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V4y)\ V1s))\wedge((ap\ V0f\ V3x) = (ap\ V0f\ V4y))))\Rightarrow(V3x = V4y))))))\Rightarrow((\exists V5g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(p\ (ap\ (ap\ c\_2Ereal\_\_topology\_2Ehomeomorphism\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal}))\ V1s)\ V2t))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})\ (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}))\ V0f)\ V5g))))\Leftrightarrow(\forall V6u \in (2^{ty\_2Erealax\_2Ereal}).((p\ (ap\ (ap\ (\mathsf{c\_2Etopology\_2Eclosed\_\_in}\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean})\ V1s))\ V6u))\Rightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Etopology\_2Eclosed\_\_in}\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean})\ V2t))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V0f)\ V6u)))))))))) \tag{25}$$

Theorem 1

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V2t \in (2^{ty\_2Erealax\_2Ereal}). (((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econtinuous\_\_on}\ V0f)\ V1s))\wedge (((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V0f)\ V1s) = V2t)\wedge(\forall V3x \in ty\_2Erealax\_2Ereal.(\forall V4y \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V3x)\ V1s))\wedge((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V4y)\ V1s))\wedge((ap\ V0f\ V3x) = (ap\ V0f\ V4y))))\Rightarrow(V3x = V4y))))))\Rightarrow((\forall V5u \in (2^{ty\_2Erealax\_2Ereal}).((p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean})\ V1s))\ V5u))\Rightarrow(p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean})\ V2t))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V0f)\ V5u)))))\Leftrightarrow(\forall V6u \in (2^{ty\_2Erealax\_2Ereal}).((p\ (ap\ (ap\ (\mathsf{c\_2Etopology\_2Eclosed\_\_in}\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean})\ V1s))\ V6u))\Rightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Etopology\_2Eclosed\_\_in}\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean})\ V2t))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V0f)\ V6u))))))))))$$