

thm\_2Ereal\_topology\_2EINTERIOR\_INTER  
 (TMJACm8tG7F3epfBuZvQB8CtFeu5y9mCcpd)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_2E21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2EF$  to be  $(ap (c\_2Ebool\_2E\_2E21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_2E7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2EF$

**Definition 7** We define  $c\_2Ebool\_2E\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 8** We define  $c\_2Ebool\_2E\_2E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_2E21 2) (\lambda V2t \in 2.V2t))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (1)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (2)$$

**Definition 9** We define  $c\_2Epair\_2E\_2EC$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2E$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \quad (3)$$

**Definition 10** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (4)$$

**Definition 11** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c\_2Ereal\_topology\_2EDist : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \quad (5)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (6)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (7)$$

**Definition 12** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ **if**  $(\exists x \in A.p (ap\ P\ x))$  **then**  $(the\ (\lambda x.x \in A \wedge P\ x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 13** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E\_40 (t$

Let  $c\_2Erealax\_2Etrealm\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (8)$$

**Definition 14** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (9)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (10)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (11)$$

**Definition 15** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (12)$$

**Definition 16** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 17** We define  $c\_2Ereal\_topology\_2EOpen$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap (c\_2Ebool\_2E2$

**Definition 18** We define  $c\_2Ereal\_topology\_2Einterior$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap (c\_2Epred\_2E2$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (15) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow \neg (p V0t)))))) \quad (16) \end{aligned}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg (p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg ( \\ & p V0t)))))) \quad (18) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (19) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in \\ & 2.(((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27)))))) \quad (20) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in \\ & (2^{A\_27a}).(((p (ap (ap (c\_2Epred\_set\_2ESUBSET A\_27a) V0s) V1t)) \wedge \\ & (p (ap (ap (c\_2Epred\_set\_2ESUBSET A\_27a) V1t) V0s))) \Rightarrow (V0s = V1t)))) \quad (21) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\ & (2^{A.27a}).(\forall V2x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool\_2EIN\ A.27a) \\ & V2x)\ (ap\ (ap\ (c.2Epred\_set\_2EINTER\ A.27a)\ V0s)\ V1t)))) \Leftrightarrow ((p\ (ap\ (22) \\ & (ap\ (c.2Ebool\_2EIN\ A.27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c.2Ebool\_2EIN \\ & A.27a)\ V2x)\ V1t)))))) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0s \in (2^{A.27a}).(\forall V1t \in \\ & (2^{A.27a}).(p\ (ap\ (ap\ (c.2Epred\_set\_2ESUBSET\ A.27a)\ (ap\ (ap\ (c.2Epred\_set\_2EINTER \\ & A.27a)\ V0s)\ V1t))\ V0s)))) \wedge (\forall V2s \in (2^{A.27a}).(\forall V3t \in \\ & (2^{A.27a}).(p\ (ap\ (ap\ (c.2Epred\_set\_2ESUBSET\ A.27a)\ (ap\ (ap\ (c.2Epred\_set\_2EINTER \\ & A.27a)\ V3t)\ V2s))\ V2s)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\ & (2^{A.27a}).(\forall V2u \in (2^{A.27a}).((p\ (ap\ (ap\ (c.2Epred\_set\_2ESUBSET \\ & A.27a)\ V0s)\ (ap\ (ap\ (c.2Epred\_set\_2EINTER\ A.27a)\ V1t)\ V2u)))) \Leftrightarrow (24) \\ & ((p\ (ap\ (ap\ (c.2Epred\_set\_2ESUBSET\ A.27a)\ V0s)\ V1t)) \wedge (p\ (ap\ (ap\ (c.2Epred\_set\_2ESUBSET\ A.27a)\ V0s)\ V2u)))))) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1t \in (2^{ty\_2Erealax\_2Ereal}). \\ & (((p\ (ap\ c.2Ereal\_topology\_2EOpen\ V0s)) \wedge (p\ (ap\ c.2Ereal\_topology\_2EOpen \\ & V1t)))) \Rightarrow (p\ (ap\ c.2Ereal\_topology\_2EOpen\ (ap\ (ap\ (c.2Epred\_set\_2EINTER \\ & ty\_2Erealax\_2Ereal)\ V0s)\ V1t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(p\ (ap\ c.2Ereal\_topology\_2EOpen\ (ap\ c.2Ereal\_topology\_2Einterior\ V0s)))) \quad (26)$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(p\ (ap\ (ap\ (c.2Epred\_set\_2ESUBSET\ ty\_2Erealax\_2Ereal)\ (ap\ c.2Ereal\_topology\_2Einterior\ V0s))\ V0s))) \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1t \in (2^{ty\_2Erealax\_2Ereal}). \\ & ((p\ (ap\ (ap\ (c.2Epred\_set\_2ESUBSET\ ty\_2Erealax\_2Ereal)\ V0s)\ V1t)) \Rightarrow (p\ (ap\ (ap\ (c.2Epred\_set\_2ESUBSET\ ty\_2Erealax\_2Ereal) \\ & (ap\ c.2Ereal\_topology\_2Einterior\ V0s))\ (ap\ c.2Ereal\_topology\_2Einterior \\ & V1t)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1t \in (2^{ty\_2Erealax\_2Ereal}). \\
& (((p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) V1t) \\
V0s)) \wedge (p (ap c\_2Ereal\_topology\_2EOpen V1t)))) \Rightarrow (p (ap (ap (c\_2Epred\_set\_2ESUBSET \\
& ty\_2Erealax\_2Ereal) V1t) (ap c\_2Ereal\_topology\_2Einterior \\
& V0s))))))
\end{aligned} \tag{29}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1t \in (2^{ty\_2Erealax\_2Ereal}). \\
& ((ap c\_2Ereal\_topology\_2Einterior (ap (ap (c\_2Epred\_set\_2EINTER \\
& ty\_2Erealax\_2Ereal) V0s) V1t)) = (ap (ap (c\_2Epred\_set\_2EINTER \\
& ty\_2Erealax\_2Ereal) (ap c\_2Ereal\_topology\_2Einterior V0s)) \\
& (ap c\_2Ereal\_topology\_2Einterior V1t))))))
\end{aligned}$$