

thm_2Ereal__topology_2EIS__INTERVAL__IMP__LOCALLY__COM
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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_2E21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2EF$ to be $(ap (c_2Ebool_2E_2E21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2E7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2EF$

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \tag{1}$$

Definition 7 We define $c_2Ebool_2E_2EIN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A-27a}).(ap V1f V0x)))$

Definition 8 We define $c_2Ebool_2E_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_2E21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A-27b})^{A-27a}}) \tag{3}$$

Definition 9 We define $c_2Epair_2E_2EC$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (4)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (5)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (6)$$

Definition 10 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p (ap\ P\ x))$ then $(the\ (\lambda x.x \in A \wedge p\ x))$ of type $\iota \Rightarrow \iota$.

Definition 11 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal$. $(ap\ (c_2Emin_2E40\ (ap\ V0a)))$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (7)$$

Definition 12 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal$. $\lambda V1T2 \in ty_2Erealax_2Ereal$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (8)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (10)$$

Definition 13 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (11)$$

Definition 14 We define c_2Ebool_2E3F to be $\lambda A.\lambda 27a : \iota$. $(\lambda V0P \in (2^{A-27a})$. $(ap\ V0P\ (ap\ (c_2Emin_2E40\ (ap\ V0P))))$)

Definition 15 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal})$. $(ap\ (c_2Ebool_2E27a\ (ap\ V0s)))$

Let $c_2Ereal_topology_2Eball : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2Eball \in ((2^{ty_2Erealax_2Ereal})^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (12)$$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda 27a.nonempty\ A.\lambda 27b.nonempty\ A.\lambda 27b \Rightarrow c_2Epred_set_2EGSPEC\ A.\lambda 27a\ A.\lambda 27b \in ((2^{A-27a})^{(ty_2Epair_2Eprod\ A.\lambda 27a\ 2)^{A-27b}}) \quad (13)$$

Definition 16 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Epred_set_2EINTER) s t)$.
Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (14)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (15)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (16)$$

Let $c_2Elist_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EHD A_27a \in (A_27a^{(ty_2Elist_2Elist A_27a)}) \quad (17)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (18)$$

Definition 17 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal. \lambda V1y \in ty_2Erealax_2Ereal$.

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (19)$$

Definition 18 We define $c_2Ereal_topology_2ECLOSED_interval$ to be $\lambda V0l \in (ty_2Elist_2Elist (ty_2Epair_2Eprod))$.

Definition 19 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2. (c_2Ebool_2E_5C_2F t1 t2))))$.

Definition 20 We define $c_2Ereal_topology_2Eis_interval$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}). (ap (c_2Ebool_2E_5C_2F) s)$.

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Etopology_2Etopology A0) \quad (20)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Etopology_2Etopology A_27a \in ((ty_2Etopology_2Etopology A_27a)^{(2^{(2^{A_27a})})}) \quad (21)$$

Definition 21 We define $c_2Ereal_topology_2Eeuclidean$ to be $(ap (c_2Etopology_2Etopology ty_2Erealax_2Ereal))$.

Let $c_2Etopology_2Eopen_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Eopen_in\ A_27a \in ((2^{A_27a})^{(ty_2Etopology_2Etopology\ A_27a)}) \quad (22)$$

Definition 22 We define $c_2Ereal_topology_2Esubtopology$ to be $\lambda A_27a : \iota.\lambda V0top \in (ty_2Etopology_2Etopology\ A_27a)$

Definition 23 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Ereal_topology_2Esubtopology\ A_27a)\ s)\ t$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \quad (23)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \quad (24)$$

Definition 24 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Definition 25 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(c_2Eprim_rec_2E_3C\ m\ n)$

Definition 26 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(c_2Earithmetic_2E_3E\ m\ n)$

Definition 27 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(c_2Earithmetic_2E_3E_3D\ m\ n)$

Let $ty_2Ereal_topology_2Enet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ereal_topology_2Enet\ A0) \quad (25)$$

Let $c_2Ereal_topology_2Emk_net : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ereal_topology_2Emk_net\ A_27a \in ((ty_2Ereal_topology_2Enet\ A_27a)^{(2^{A_27a})^{A_27a}}) \quad (26)$$

Definition 28 We define $c_2Ereal_topology_2Esequentially$ to be $(ap\ (c_2Ereal_topology_2Emk_net\ ty_2Ereal_topology_2Enet))$

Definition 29 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g \in (A_27c^{A_27a}).(c_2Ecombin_2Eo\ f\ g)$

Let $c_2Ereal_topology_2Enetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ereal_topology_2Enetord\ A_27a \in (((2^{A_27a})^{A_27a})^{(ty_2Ereal_topology_2Enet\ A_27a)}) \quad (27)$$

Definition 30 We define $c_2Ereal_topology_2Etrivial_limit$ to be $\lambda A_27a : \iota.\lambda V0net \in (ty_2Ereal_topology_2Enet\ A_27a)$

Definition 31 We define $c_2Ereal_topology_2Eeventually$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1net \in (ty_2Ereal_topology_2Enet\ A_27a)$

Definition 32 We define $c_2Ereal_topology_2E_2D_2D_3E$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal\ A_27a)$

Definition 33 We define $c_2Ereal_topology_2Ecompact$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (c_2Ebool_2Eand\ s))$

Definition 34 We define $c_2Ereal_topology_2Elocally$ to be $\lambda V0P \in (2^{(2^{ty_2Erealax_2Ereal})}).\lambda V1s \in (2^{ty_2Ere$

Assume the following.

$$True \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V0t)))) \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (30) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (31) \end{aligned}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p V0t)))))) \quad (33) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (34) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\ & 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \quad (35) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in \\ & 2.(((\exists V2x \in A_27a.(p (ap V0P V2x))) \Rightarrow (p V1Q)) \Leftrightarrow (\forall V3x \in \\ & A_27a.((p (ap V0P V3x)) \Rightarrow (p V1Q)))))) \quad (36) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A-27a}). (\forall V1t \in \\ & (2^{A-27a}). (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a) \\ & V2x)\ (ap\ (ap\ (c.2Epred_set.2EINTER\ A.27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (37) \\ & (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c.2Ebool.2EIN \\ & A.27a)\ V2x)\ V1t)))))) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty.2Erealx.2Ereal. (\forall V1e \in ty.2Erealx.2Ereal. \\ & (p\ (ap\ c.2Ereal_topology.2EOpen\ (ap\ c.2Ereal_topology.2Eball \\ & (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealx.2Ereal\ ty.2Erealx.2Ereal) \\ & V0x)\ V1e)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & ((\forall V0x \in ty.2Erealx.2Ereal. (\forall V1e \in ty.2Erealx.2Ereal. \\ & ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ ty.2Erealx.2Ereal)\ V0x)\ (ap\ c.2Ereal_topology.2Eball \\ & (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealx.2Ereal\ ty.2Erealx.2Ereal) \\ & V0x)\ V1e)))) \Leftrightarrow (p\ (ap\ (ap\ c.2Erealx.2Ereal_lt\ (ap\ c.2Ereal.2Ereal_of_num \\ & c.2Enum.2E0)\ V1e)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & (\forall V0u \in (2^{ty.2Erealx.2Ereal}). (\forall V1s \in (2^{ty.2Erealx.2Ereal}). \\ & ((p\ (ap\ c.2Ereal_topology.2EOpen\ V1s)) \Rightarrow (p\ (ap\ (ap\ (c.2Etopology.2Eopen_in \\ & ty.2Erealx.2Ereal)\ (ap\ (ap\ (c.2Ereal_topology.2Esubtopology \\ & ty.2Erealx.2Ereal)\ c.2Ereal_topology.2Eeuclidean)\ V0u)) \\ & (ap\ (ap\ (c.2Epred_set.2EINTER\ ty.2Erealx.2Ereal)\ V0u)\ V1s)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty.2Erealx.2Ereal. (\forall V1b \in ty.2Erealx.2Ereal. \\ & (p\ (ap\ c.2Ereal_topology.2Ecompact\ (ap\ c.2Ereal_topology.2ECLOSED_interval \\ & (ap\ (ap\ (c.2Elist.2ECONS\ (ty.2Epair.2Eprod\ ty.2Erealx.2Ereal \\ & ty.2Erealx.2Ereal))\ (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealx.2Ereal \\ & ty.2Erealx.2Ereal)\ V0a)\ V1b))\ (c.2Elist.2ENIL\ (ty.2Epair.2Eprod \\ & ty.2Erealx.2Ereal\ ty.2Erealx.2Ereal)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1x \in ty_2Erealax_2Ereal. \\
& (((p (ap c_2Ereal_topology_2Eis_interval V0s)) \wedge (p (ap (ap (\\
& c_2Ebool_2EIN ty_2Erealax_2Ereal) V1x) V0s))) \Rightarrow (\exists V2a \in \\
& ty_2Erealax_2Ereal. (\exists V3b \in ty_2Erealax_2Ereal. (\exists V4d \in \\
& ty_2Erealax_2Ereal. ((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V4d)) \wedge ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V1x) (ap c_2Ereal_topology_2ECLOSED_interval (ap (ap (c_2Elist_2ECONS \\
& (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)) \\
& (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V2a) V3b)) (c_2Elist_2ENIL (ty_2Epair_2Eprod ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal)))))) \wedge ((p (ap (ap (c_2Epred_set_2ESUBSET \\
& ty_2Erealax_2Ereal) (ap c_2Ereal_topology_2ECLOSED_interval \\
& (ap (ap (c_2Elist_2ECONS (ty_2Epair_2Eprod ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal)) (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal) V2a) V3b)) (c_2Elist_2ENIL (ty_2Epair_2Eprod \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal)))))) V0s)) \wedge (p (ap (ap \\
& (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) (ap (ap (c_2Epred_set_2EINTER \\
& ty_2Erealax_2Ereal) (ap c_2Ereal_topology_2Eball (ap (ap (c_2Epair_2E_2C \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V1x) V4d))) V0s)) (ap \\
& c_2Ereal_topology_2ECLOSED_interval (ap (ap (c_2Elist_2ECONS \\
& (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)) \\
& (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V2a) V3b)) (c_2Elist_2ENIL (ty_2Epair_2Eprod ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal)))))))))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).((p (ap (ap c_2Ereal_topology_2Ellocally \\
& c_2Ereal_topology_2Ecompact) V0s)) \Leftrightarrow (\forall V1x \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V1x) V0s)) \Rightarrow (\exists V2u \in \\
& (2^{ty_2Erealax_2Ereal}). (\exists V3v \in (2^{ty_2Erealax_2Ereal}). \\
& ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V1x) V2u)) \wedge ((p \\
& (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) V2u) V3v)) \wedge \\
& ((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) V3v) \\
& V0s)) \wedge ((p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) \\
& (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& c_2Ereal_topology_2Euclidean) V0s)) V2u)) \wedge (p (ap c_2Ereal_topology_2Ecompact \\
& V3v)))))))))))))
\end{aligned} \tag{43}$$

Theorem 1

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).((p (ap c_2Ereal_topology_2Eis_interval \\
& V0s)) \Rightarrow (p (ap (ap c_2Ereal_topology_2Ellocally c_2Ereal_topology_2Ecompact) \\
& V0s))))
\end{aligned}$$