thm_2Ereal__topology_2EJOINABLE__COMPONENTS__EQ (TM-PDwafhKFD16nVq68LACZdMZXYBdNnufhT)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 4 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{2}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{3}$$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27b}$

Definition 7 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 8 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2EF})$.

Definition 9 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 10 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 11 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{4}$$

Definition 12 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{5}$$

Definition 13 *We define* c_2Epred__set_2EINTER *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_$

Definition 14 *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ ($

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{6}$$

Definition 15 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 16 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 17 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c.$

Let $c\_2Ereal\_\_topology\_2EDist : \iota$ be given. Assume the following.

$$c\_2Ereal\_\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \tag{7}$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{8}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{9}$$

Definition 18 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (\mathsf{c\_2Emin\_2E\_40}\ (t$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehr} \quad (10)$$

Definition 19 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \quad (11)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (12)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (13)$$

Definition 20 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (14)$$

Definition 21 *We define* c_2Ereal__topology_2EOpen *to be* $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ ($c_2Ebool_2E_2

Definition 22 *We define* c_2Ereal__topology_2Econnected *to be* $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap$ c_2Ebool_2

Definition 23 *We define* c_2Ereal__topology_2Econnected__component *to be* $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).\lambda V$

Definition 24 *We define* c_2Ereal__topology_2Ecomponents *to be* $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap\ (c\_2Epre$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg (p\ V0t)))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.((\neg (p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \quad (19)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t)))\wedge(((\neg True)\Leftrightarrow False)\wedge ((\neg False)\Leftrightarrow True))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y)\Leftrightarrow(V1y = V0x)))) \tag{21}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow (p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).((\neg(\forall V1x \in A\_27a.(p\ (ap\ V0P\ V1x))))\Leftrightarrow(\exists V2x \in A\_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).((\exists V2x \in A\_27a.((p\ V0P)\wedge(p\ (ap\ V1Q\ V2x))))\Leftrightarrow((p\ V0P)\wedge(\exists V3x \in A\_27a.(p\ (ap\ V1Q\ V3x))))))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ V0P)\Rightarrow(p\ (ap\ V1Q\ V2x))))\Leftrightarrow((p\ V0P)\Rightarrow(\forall V3x \in A\_27a.(p\ (ap\ V1Q\ V3x))))))) \tag{25}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A)\vee((p\ V1B)\vee(p\ V2C)))\Leftrightarrow(((p\ V0A)\vee(p\ V1B))\vee(p\ V2C)))))) \tag{26}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A)\vee(p\ V1B))\Leftrightarrow((p\ V1B)\vee (p\ V0A))))) \tag{27}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\wedge(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\vee(\neg(p\ V1B))))\wedge((\neg((p\ V0A)\vee(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\wedge(\neg(p\ V1B))))))) \tag{28}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow ((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{29}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow(((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))) \tag{30}$$

Assume the following.

$$(\forall V0r \in 2.(\forall V1p \in 2.(\forall V2q \in 2.((((p\ V1p)\wedge(p\ V2q))\Rightarrow(p\ V0r))\Leftrightarrow((p\ V1p)\Rightarrow((p\ V2q)\Rightarrow(p\ V0r))))))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27c.nonempty\ A\_27c\Rightarrow\forall A\_27d.nonempty\ A\_27d\Rightarrow\forall A\_27e.nonempty\ A\_27e\Rightarrow\forall A\_27f.nonempty\ A\_27f\Rightarrow\forall A\_27g.nonempty\ A\_27g\Rightarrow(\forall V0Q \in (2^{A\_27b}).((\forall V1P \in (2^{A\_27a}).(\forall V2f \in (A\_27b^{A\_27a}).((\forall V3z \in A\_27b.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V3z)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27b\ A\_27a)\ (\lambda V4x \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27b\ 2)\ (ap\ V2f\ V4x))\ (ap\ V1P\ V4x))))))\Rightarrow(p\ (ap\ V0Q\ V3z))))\Leftrightarrow(\forall V5x \in A\_27a.((p\ (ap\ V1P\ V5x))\Rightarrow(p\ (ap\ V0Q\ (ap\ V2f\ V5x))))))))\wedge((\forall V6P \in ((2^{A\_27d})^{A\_27c}).(\forall V7f \in ((A\_27b^{A\_27d})^{A\_27c}).((\forall V8z \in A\_27b.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V8z)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27b\ (ty\_2Epair\_2Eprod\ A\_27c\ A\_27d))\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27c\ A\_27d\ (ty\_2Epair\_2Eprod\ A\_27b\ 2))\ (\lambda V9x \in A\_27c.(\lambda V10y \in A\_27d.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27b\ 2)\ (ap\ (ap\ V7f\ V9x)\ V10y))\ (ap\ (ap\ V6P\ V9x)\ V10y)))))))))\Rightarrow(p\ (ap\ V0Q\ V8z))))\Leftrightarrow(\forall V11x \in A\_27c.(\forall V12y \in A\_27d.((p\ (ap\ (ap\ V6P\ V11x)\ V12y))\Rightarrow(p\ (ap\ V0Q\ (ap\ (ap\ V7f\ V11x)\ V12y)))))))))\wedge(\forall V13P \in (((2^{A\_27g})^{A\_27f})^{A\_27e}).(\forall V14f \in (((A\_27b^{A\_27g})^{A\_27f})^{A\_27e}).((\forall V15z \in A\_27b.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V15z)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27b\ (ty\_2Epair\_2Eprod\ A\_27e\ (ty\_2Epair\_2Eprod\ A\_27f\ A\_27g)))\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27e\ (ty\_2Epair\_2Eprod\ A\_27f\ A\_27g)\ (ty\_2Epair\_2Eprod\ A\_27b\ 2))\ (\lambda V16w \in A\_27e.(ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27f\ A\_27g\ (ty\_2Epair\_2Eprod\ A\_27b\ 2))\ (\lambda V17x \in A\_27f.(\lambda V18y \in A\_27g.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27b\ 2)\ (ap\ (ap\ (ap\ V14f\ V16w)\ V17x)\ V18y))\ (ap\ (ap\ (ap\ V13P\ V16w)\ V17x)\ V18y))))))))))\Rightarrow(p\ (ap\ V0Q\ V15z))))\Leftrightarrow(\forall V19w \in A\_27e.(\forall V20x \in A\_27f.(\forall V21y \in A\_27g.((p\ (ap\ (ap\ (ap\ V13P\ V19w)\ V20x)\ V21y))\Rightarrow(p\ (ap\ V0Q\ (ap\ (ap\ (ap\ V14f\ V19w)\ V20x)\ V21y)))))))))))) \tag{32}$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1t \in (2^{ty\_2Erealax\_2Ereal}). (\forall V2x \in ty\_2Erealax\_2Ereal.(\forall V3y \in ty\_2Erealax\_2Ereal. (((p\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econnected}\ V1t))\wedge((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ ty\_2Erealax\_2Ereal)\ V1t)\ V0s))\wedge((\neg((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econnected\_\_component}\ V0s)\ V2x))\ V1t) = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Erealax\_2Ereal)))\wedge (\neg((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econnected\_\_component}\ V0s)\ V3y))\ V1t) = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Erealax\_2Ereal))))))\Rightarrow((ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econnected\_\_component}\ V0s)\ V2x) = (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econnected\_\_component}\ V0s)\ V3y)))))))$$ (33)

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{34}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{35}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow (((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{36}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow ((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{37}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{38}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee ((\neg(p\ V1q))\vee(\neg(p\ V0p))))))))))) \tag{39}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee (\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{40}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{41}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{42}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{43}$$

Theorem 1

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1t \in (2^{ty\_2Erealax\_2Ereal}).(\forall V2c1 \in (2^{ty\_2Erealax\_2Ereal}).(\forall V3c2 \in (2^{ty\_2Erealax\_2Ereal}).((((p\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econnected}\ V1t))\wedge((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ ty\_2Erealax\_2Ereal)\ V1t)\ V0s))\wedge((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{ty\_2Erealax\_2Ereal}))\ V2c1)\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Ecomponents}\ V0s)))\wedge((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{ty\_2Erealax\_2Ereal}))\ V3c2)\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Ecomponents}\ V0s)))\wedge((\neg((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ ty\_2Erealax\_2Ereal)\ V2c1)\ V1t) = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Erealax\_2Ereal)))\wedge(\neg((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ ty\_2Erealax\_2Ereal)\ V3c2)\ V1t) = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Erealax\_2Ereal))))))))\Rightarrow(V2c1 = V3c2))))))$$